Chapter 1

A Unifying Framework for Objective Criteria Weighting in Multi-Criteria Decision Making: Normalisation Invariance, Degeneracy and Selection

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Abstract

Objective weighting methods derive criterion importances directly from a decision matrix, without eliciting preferences from a decision maker. A large and growing family of such methods is now in routine use, yet they are often studied empirically (applied to a dataset and compared through the correlation of the resulting weights) or surveyed descriptively. This chapter takes a different, analytical route. It places seven widely used objective methods (the standard deviation method, the coefficient of variation, the Shannon entropy weight method, CRITIC, MEREC, LOPCOW and CILOS, together with the composite IDOCRIW) inside a single three-stage decomposition: a normalisation, a per-criterion dispersion or information functional and an optional de-correlation or structural adjustment, followed by a sum-to-one normalisation. Within this framework two structural properties are established with short proofs and counterexamples. First, a normalisation-invariance result classifies which methods return identical weights when the input is sum-, vector- or max-normalised: the entropy, coefficient-of-variation, LOPCOW, MEREC and CILOS weights are invariant to this choice, whereas the standard deviation and CRITIC weights are not in general, because their dispersion term scales with the column. Second, a degeneracy result identifies the inputs on which each method returns a zero weight, an undefined weight or fails to be well-defined; in particular, under the stated assumptions, a constant criterion receives zero weight in SD, CV, EWM and MEREC, whereas CRITIC, LOPCOW and CILOS are not well-defined. A worked example illustrates both results and a further contrast: the methods can encode different notions

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of what "important" means, so the same near-constant criterion can receive either the smallest or the largest weight. The analysis yields a property-based taxonomy and a compact reporting-and-selection protocol for applied work.

1. Introduction

Most multi-criteria decision-making (*MCDM*) procedures require a vector of criterion weights before alternatives can be aggregated and ranked. Weights can be supplied subjectively, by eliciting judgements from a decision maker, or objectively, by extracting them from the decision matrix itself. Objective weighting is attractive when subjective elicitation is impractical, when reproducibility is valued or when the analyst wishes to reduce the role of subjective preference input. A steady stream of objective methods has appeared, from the classical standard deviation (*SD*) and Shannon entropy approaches to the CRiteria Importance Through Intercriteria Correlation (*CRITIC*) method (Diakoulaki, Mavrotas and Papayannakis, 1995), the MEthod based on the Removal Effects of Criteria (*MEREC*) (Keshavarz-Ghorabaee, Amiri, Zavadskas, Turskis and Antucheviciene, 2021), the LOgarithmic Percentage Change-driven Objective Weighting (*LOPCOW*) method (Ecer and Pamucar, 2022) and the Criterion Impact LOSs (*CILOS*) method with its composite extension, the Integrated Determination of Objective CRIteria Weights (*IDOCRIW*) (Zavadskas and Podvezko, 2016), among many others.

A substantial part of the literature on these methods can be grouped into two broad genres. The first is descriptive review: methods are listed, their steps summarised and their strengths and weaknesses tabulated (Odu, 2019; Ayan, Abacıoğlu and Basilio, 2023). The second is empirical comparison: several methods are applied to one or more decision matrices, and the resulting weight vectors, or the rankings they produce, are compared through correlation coefficients (Paradowski, Bączkiewicz and Wątróbski, 2021; Mukhametzyanov, 2021). Both genres are valuable, but both mainly assess the methods through their outputs. Less common is a structural account that explains, before any data are seen, why the methods differ and when those differences appear.

This chapter provides such an account. Its first contribution is a unifying decomposition that writes the per-criterion methods considered here as the same three-stage pipeline: a normalisation, then a per-criterion dispersion or information functional, then an optional structural de-correlation adjustment, followed by a sum-to-one step. This decomposition identifies *CILOS* and its composite *IDOCRIW* as the system-based structural class that sits at the boundary of that pipeline. Placing heterogeneous methods in one form makes their similarities and differences explicit and allows several observations to be stated under explicit assumptions.

The second contribution is two analytical results within that framework. The normalisation-invariance result addresses a recurring question that is usually answered method by method and only empirically: under which normalisations do the weights stay the same? It is shown that the entropy, coefficient-of-variation, *LOPCOW*, *MEREC* and *CILOS* weights are invariant to the choice among the sum, vector and max normalisations, while the *SD* and *CRITIC* weights are not in general. The degeneracy result characterises the inputs on which each method returns a zero weight, an undefined weight or is not well-defined; the practically important case is a constant or near-constant criterion, which receives zero weight in *SD*, *CV*, *EWM* and *MEREC* unless an explicit diagnostic is reported, but is not well-defined for *CRITIC*, *LOPCOW* and *CILOS*. A worked example with generated numbers illustrates both results and the more basic point that the methods encode different and sometimes opposite notions of importance.

The third contribution is practical: a property-based taxonomy and a compact reporting-and-selection protocol. The protocol's central recommendation follows directly from the invariance result: because some methods are sensitive to the normalisation while others are not, the normalisation actually used should be reported with the method, and a method should be chosen with its invariance class and data requirements in mind.

The treatment is deliberately analytical and not based on an empirical case study; the single example uses invented numbers and is used only to illustrate the analytical results. The chapter therefore complements, rather than repeats, the empirical and bibliometric literature on weighting.

The remainder is organised as follows. Section 2 fixes notation and the normalisations. Section 3 develops the unifying decomposition. Section 4 catalogues the methods. Sections 5 and 6 establish the invariance and degeneracy results. Section 7 gives the worked illustration. Section 8 presents the taxonomy and Section 9 the reporting-and-selection protocol. Section 10 concludes.

2. The Objective Weighting Problem

A decision matrix is an array $X = \left[x_{ij} \right]$ with m alternatives in the rows ($i = 1, \dots, m$) and n criteria in the columns ($j = 1, \dots, n$). An objective weighting method is a map that takes X to a weight vector $w = \left(w_1, \dots, w_n \right)$ with

$$w_j \geq 0, \quad \sum_{j=1}^{n} w_j = 1, \tag{1}$$

using the entries of X alone. Throughout, the criteria are benefit criteria, so larger entries are preferred; cost criteria are handled by a prior orientation step and do not affect the structural arguments below. The j-th column is written $x_{\cdot j}$, with column mean μ_j and population standard deviation σ_j. Table 1 collects the symbols used in the chapter.

Table 1. Notation and symbols.

Symbol	Meaning
$X = \left[x_{ij}\right]$	decision matrix, m alternatives by n criteria
$m,\ n$	number of alternatives; number of criteria
$x_{\cdot j}$	the j-th criterion column
$\mu_j,\ \sigma_j$	mean and standard deviation of column j
ρ_{jk}	linear correlation between columns j and k
$R = \left[r_{ij}\right]$	normalised decision matrix
w_j	weight of criterion j, with $w_j \geq 0$ and $\sum_j w_j = 1$
$\Delta_j,\ \Gamma_j$	dispersion functional; structural factor (Section 3)

Source: author's compilation.

Because the criteria are generally measured on different scales, the columns are first made comparable by a normalisation that replaces X by $R = \left[r_{ij}\right]$, column by column. Four normalisations recur in the literature and are used here (Aytekin, 2021); they are listed in Table 2. The sum, vector and max normalisations are

$$r_{ij} = \frac{x_{ij}}{\sum_i x_{ij}}, \quad r_{ij} = \frac{x_{ij}}{\sqrt{\sum_i x_{ij}^2}}, \quad r_{ij} = \frac{x_{ij}}{\max_i x_{ij}}, \tag{2}$$

and the min-max normalisation is

$$r_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}. \tag{3}$$

The three former share a structural feature that drives the results of Section 5: each replaces a column by that column divided by a positive constant, that is, $r_{\cdot j} = x_{\cdot j} / s_j$ with $s_j > 0$. They are *pure-scaling* normalisations. The min-max normalisation is different: it subtracts the column minimum before scaling, so it is an *affine* (shifting) transform. This distinction, rather than the names of the individual schemes, is what matters.

Table 2. Normalisation techniques used in the chapter.

Normalisation	**Formula (benefit)**	**Type**
Sum	$x_{ij} / \sum_i x_{ij}$	pure scaling
Vector	$x_{ij} / \sqrt{\sum_i x_{ij}^2}$	pure scaling
Max	$x_{ij} / \max_i x_{ij}$	pure scaling
Min-max	$\left(x_{ij} - \min_i x_{ij}\right) / \left(\max_i x_{ij} - \min_i x_{ij}\right)$	affine (shifting)

Source: author's compilation from Aytekin (2021).

3. A Unifying Decomposition

Despite their varied motivations, most of the methods considered here share a common skeleton. Each can be written as

$$w_j = \frac{\varphi_j}{\sum_k \varphi_k}, \quad \varphi_j = \Delta_j \cdot \Gamma_j, \tag{4}$$

where Δ_j is a *dispersion or information functional* computed on a normalised column (it measures how much the alternatives differ on criterion j), and Γ_j is an optional *structural factor* that adjusts Δ_j for the relationship between criterion j and the others (typically a de-correlation or redundancy term). Methods with $\Gamma_j \equiv 1$ are *internal*: they look only at each criterion's own spread. Methods with a non-trivial Γ_j are *structural*: they also reward a criterion for carrying information not already present in the others.

This decomposition is the organising device of the chapter. It exposes two independent design choices behind each per-criterion method: the functional Δ_j that quantifies dispersion and the question of whether and how the method discounts redundancy through Γ_j. It also localises the source of each later result. The invariance behaviour of a method is governed by whether Δ_j is invariant under positive scaling of its column; the degeneracy behaviour is governed by the points at which Δ_j or Γ_j vanishes or becomes undefined. Two methods, *CILOS* and *IDOCRIW*, do not reduce to a single per-criterion functional: *CILOS* solves a homogeneous linear system built from an impact-loss matrix, and *IDOCRIW* multiplies the *CILOS* weights by entropy weights. They are therefore treated as the *system-based structural* class that sits at the boundary of the decomposition, and they are included in the invariance and degeneracy analysis even though they have no closed φ_j.

4. A Catalogue of Objective Weighting Methods

Seven methods are catalogued, each with the source from which its formula is verified. Of these, *SD*, *CV*, the entropy weight method and *LOPCOW* are *internal* ($\Gamma_j \equiv 1$), while *CRITIC*, *MEREC* and *CILOS* are *structural*, though they use different kinds of structure: *CRITIC* an explicit de-correlation factor, *MEREC* a removal effect and *CILOS* an impact-loss system.

Standard deviation (*SD*). The dispersion functional is the column standard deviation of the normalised matrix, and weights are proportional to it:

$$w_j = \frac{\sigma_j}{\sum_k \sigma_k}. \tag{5}$$

A criterion on which the alternatives vary little is treated as carrying little discriminating information.

Coefficient of variation (*CV*). To remove the influence of the column level, the standard deviation is divided by the mean before normalising:

$$w_j = \frac{\sigma_j / \mu_j}{\sum_k \sigma_k / \mu_k}. \tag{6}$$

Shannon entropy weight method (*EWM*). The column is first turned into a distribution, $p_{ij} = x_{ij} / \sum x_{ij}$, and its Shannon entropy is computed and rescaled to the unit interval[i] (Shannon, 1948; Zeleny, 1982):

$$E_j = -\frac{1}{\ln m}\sum_i p_{ij}\ln p_{ij}, \quad d_j = 1 - E_j, \quad w_j = \frac{d_j}{\sum_k d_k}. \tag{7}$$

The degree of diversification d_j is large when the column is far from uniform, so the entropy weight method, like *SD*, rewards dispersion.

CRITIC. *CRITIC* multiplies the column standard deviation by a conflict term that sums one minus the linear correlation ρ_{jk} between criterion j and every criterion k (Diakoulaki, Mavrotas and Papayannakis, 1995):

$$C_j = \sigma_j \sum_k \left(1 - \rho_{jk}\right), \quad w_j = \frac{C_j}{\sum_k C_k}. \tag{8}$$

Here $\Delta_j = \sigma_j$ and $\Gamma_j = \sum_k (1 - \rho_{jk})$: a criterion is rewarded both for spreading the alternatives and for conflicting with the others.

MEREC. *MEREC* measures how much an alternative's overall performance changes when a criterion is removed (Keshavarz-Ghorabaee et al., 2021). With the ratio normalisation $n_{ij} = \min_i x_{ij} / x_{ij}$, the overall and reduced performances are

$$S_i = \ln\left(1 + \frac{1}{n} \sum_j \left|\ln n_{ij}\right|\right), \quad S_{ij}' = \ln\left(1 + \frac{1}{n} \sum_{k \neq j} \left|\ln n_{ik}\right|\right), \tag{9}$$

and the weight is proportional to the total removal effect $E_j = \sum_i | S_{ij}' - S_i |$, normalised to sum to one. *MEREC* is structural: a criterion's importance is defined by its marginal effect on the alternatives' scores.

LOPCOW. *LOPCOW* takes the logarithm of the ratio of a column's root mean square (*RMS*) to its standard deviation, on a normalised matrix (Ecer and Pamucar, 2022):

$$\mathrm{PV}_j = \left|\ln\left(\frac{\sqrt{\frac{1}{m}\sum_i r_{ij}^2}}{\sigma_j}\right) \times 100\right|, \quad w_j = \frac{\mathrm{PV}_j}{\sum_k \mathrm{PV}_k}. \tag{10}$$

Because the ratio inside the logarithm is scale-free and is large precisely when σ_j is small, *LOPCOW* behaves very differently from *SD* and the entropy weight method, as Section 7 shows.

CILOS. The Criterion Impact LOSs method works not from a per-criterion functional but from a matrix of mutual impact losses (Zavadskas and Podvezko, 2016). After sum-normalising the matrix, let h_j denote the index of the alternative that maximises criterion j; the j-th row of the square matrix $A = \left[a_{jk} \right]$ is then the normalised row $r_{h_j,\cdot}$, so that $a_{jj} = \max_i r_{ij}$. The relative impact loss is

$$p_{ij} = \frac{a_{jj} - a_{ij}}{a_{jj}}, \quad p_{ii} = 0, \tag{11}$$

and the weights solve the homogeneous system $F q = 0$, normalised to sum to one, where F has off-diagonal entries $f_{ij} = p_{ij}$ and diagonal entries $f_{jj} = -\sum_{i \neq j} p_{ij}$. The composite *IDOCRIW* multiplies the *CILOS* weights by the entropy weights and renormalises, $w_j \propto q_j^{\text{CILOS}} w_j^{\text{EWM}}$. Both are scale-invariant through the sum normalisation, but, as Section 6 shows, *CILOS* can be sensitive near constant criteria: as a criterion approaches constancy its loss column tends to zero and its weight becomes ill-determined. Table 3 collects the catalogue.

Table 3. The catalogue in unifying-decomposition form.

Method	**Dispersion functional** Δ_j	**Structural factor** Γ_j	**Source**
SD	σ_j	1	classical
CV	σ_j / μ_j	1	classical
EWM (entropy)	$1 - E_j$	1	Shannon (1948); Zeleny (1982)
CRITIC	σ_j	$\sum_k (1 - \rho_{jk})$	Diakoulaki et al. (1995)
MEREC	removal effect E_j	structural (built in)	Keshavarz-Ghorabaee et al. (2021)
LOPCOW	$\left\| \ln\left(\text{RMS}_j / \sigma_j \right) \right\|$	1	Ecer and Pamucar (2022)
CILOS / *IDOCRIW*	impact-loss system (no closed φ_j)	structural (system)	Zavadskas and Podvezko (2016)

Source: compiled by the author from the cited texts.

5. Normalisation Invariance

A recurring but scattered observation is that some weighting methods give the same weights regardless of which normalisation is applied, while others do not. The decomposition of Section 3 turns this into a clean statement and proof.

For the invariance results, assume that every column has the positive denominator required by the chosen pure-scaling normalisation (a non-zero column sum, vector norm or maximum).

Proposition 1 (invariance under pure-scaling normalisations). *If a method determines its weights from the columns only through quantities that are invariant under positive scaling ($x_{\cdot j} \mapsto c\,x_{\cdot j},\ c > 0$), then it returns identical weights under the sum, vector and max normalisations. The entropy, coefficient-of-variation, LOPCOW, MEREC and CILOS weights (and the composite IDOCRIW) are invariant in this sense; the standard deviation and CRITIC weights are not invariant in general.*

Proof. The sum, vector and max normalisations each replace column j by $x_{\cdot j} / s_j$ for a positive constant s_j. If a method's weights depend on each column only through scale-invariant quantities, then this replacement leaves them unchanged, so the three normalisations give the same weights. For the entropy weight method, $p_{ij} = \left(c\,x_{ij}\right) / \sum_i (c\,x_{ij}) = x_{ij} / \sum_i x_{ij}$, so the weights do not see the scale. For *CV*, $\sigma\left(c\,x\right) = c\,\sigma\left(x\right)$ and $\mu\left(c\,x\right) = c\,\mu\left(x\right)$, so σ / μ is unchanged. For *LOPCOW*, the root mean square and the standard deviation both scale by c, so their ratio, and hence PV_j, is unchanged. *CILOS* sum-normalises internally, so any prior positive column scaling cancels. *MEREC* applies its own ratio normalisation, which is itself scale-invariant, so the prior choice of normalisation does not affect it; and *IDOCRIW*, the renormalised product of the scale-invariant entropy and *CILOS* weights, inherits their invariance. By contrast $\sigma\left(c\,x\right) = c\,\sigma\left(x\right)$ is not scale-invariant, so for *SD* the ratio $\sigma_j / \sum_k \sigma_k$ depends on the per-column factors s_j, which differ across the three normalisations; the same holds for the σ_j in *CRITIC*. Hence *SD* and *CRITIC* weights change with the normalisation. □

A small but useful corollary links two of the methods.

Proposition 2 (*SD* and *CV* coincide under sum normalisation). *Under sum normalisation every column satisfies $\sum_i r_{ij} = 1$, so $\mu_j = 1/m$ for all j; therefore $\mathrm{CV}_j = \sigma_j / \mu_j = m\,\sigma_j$ and the CV and SD weights are identical.*

Proof. With $\mu_j = 1/m$ for every j, $w_j^{\mathrm{CV}} = m\sigma_j / \sum_k m\sigma_k = \sigma_j / \sum_k \sigma_k = w_j^{\mathrm{SD}}$. ▫

The affine min-max normalisation behaves differently, because it shifts the column before scaling.

Proposition 3 (sensitivity to shifting normalisations). *Methods that depend on the column through sum-normalisation, ratios or logarithms are not invariant under affine normalisations such as min-max. In particular, the entropy weight method is not scale-invariant under min-max and requires the standard convention* $0\ln 0 = 0$ *to handle the zero that min-max introduces.*

Proof. A shift $x_{ij} \mapsto x_{ij} + b$ changes $\sum_i (x_{ij} + b)$ and hence the distribution p_{ij} used by the entropy weight method, and changes μ_j used by *CV*, so neither is shift-invariant. Under min-max the column minimum maps to $r = 0$; with the standard convention $0\ln 0 = 0$ the entropy weight method remains evaluable, but because min-max is an affine transform it changes the distribution p_{ij}, so the method is not scale-invariant under it. ▫

Table 4 records the resulting invariance classes. The practical reading is immediate: for *SD* and *CRITIC* the weight vector is a joint product of the method and the normalisation, so the two must be reported together; for the invariant methods the normalisation choice among the pure-scaling schemes is immaterial.

Table 4. Normalisation-invariance of the catalogue (✓ = weights identical, ✗ = weights change).

Method	sum ↔ vector ↔ max	min-max (affine)
SD	✗ change	✗ change
CV	✓ invariant	✗ change
EWM (entropy)	✓ invariant	✗ change (needs 0·ln 0)
CRITIC	✗ change	✗ change
MEREC	✓ invariant (own ratio normalisation)	not applicable
LOPCOW	✓ invariant	✗ change
CILOS	✓ invariant (own sum normalisation)	✗ change

Source: author's analysis (Propositions 1-3), confirmed numerically in Section 7.

6. Degeneracy

Objective methods can also fail or behave trivially on particular inputs. Because the weights are ratios of the functionals in Section 3 or solutions of a homogeneous system in the case of *CILOS*, the failures occur at points where a functional vanishes or the system loses rank.

Proposition 4 (degeneracy on a constant criterion and on non-positive data). *Let criterion j be constant across alternatives, so $\sigma_j = 0$. Then, provided at least one other criterion has positive dispersion, SD, CV, the entropy weight method and MEREC assign $w_j = 0$, whereas CRITIC, LOPCOW and CILOS are not well-defined. Moreover the entropy weight method requires non-negative entries and a positive column sum (with $0 \ln 0 = 0$), MEREC requires positive entries, CILOS requires positive impact-loss denominators, LOPCOW requires only a non-zero standard deviation rather than positive raw data and CV requires a non-zero column mean (positive when the weights must be non-negative).*

Proof. If $\sigma_j = 0$ then *SD*'s numerator is zero and $w_j = 0$; the entropy weight method's column is uniform, so E_j attains its maximum 1, $d_j = 0$ and $w_j = 0$; *CV*'s numerator $\sigma_j / \mu_j = 0$; and *MEREC*'s removal effect for a criterion that does not vary is zero. For *CRITIC*, the correlation ρ_{jk} between a constant column and any other is the indeterminate form $0/0$, so Γ_j and hence C_j are undefined. For *LOPCOW*, $\sigma_j = 0$ makes the ratio $\mathrm{RMS}_j / \sigma_j$ divide by zero, so PV_j is undefined. For *CILOS*, a constant column makes every impact loss p_{ij} in that column zero, so the corresponding column of F is zero and the system $Fq = 0$ has a free coordinate: the weight of the constant criterion is indeterminate, and as a criterion only approaches constancy its *CILOS* weight becomes ill-conditioned. The entropy weight method takes logarithms of the column distribution, so it requires non-negative entries and a positive column sum (with $0 \ln 0 = 0$ for zero entries); *MEREC* takes logarithms of ratios and so requires positive entries; *CILOS* uses no logarithm but needs positive denominators in its impact-loss ratios, so it is not applied to constant criteria; *LOPCOW*'s logarithm acts on the scale-free ratio $\mathrm{RMS}_j / \sigma_j$ and so needs only a non-zero standard deviation; and *CV* divides by μ_j, so it requires a non-zero column mean (positive when the weights must be non-negative). ▫

The consequence is a screening rule rather than a curiosity. A constant or near-constant criterion is common in practice. Under the stated conditions *SD*, *CV*, the entropy weight method and *MEREC* return a zero or negligible contribution for it, but *CRITIC*, *LOPCOW* and *CILOS* should be applied only after such criteria are removed or repaired; the entropy weight method requires non-negative entries and a positive column sum, *MEREC* requires positive entries and *CILOS* requires positive impact-loss denominators. Table 5 summarises the screening requirements.

Table 5. Degeneracy and data requirements.

Method	**Constant criterion** ($\sigma_j = 0$)	**Requires positive data**	**Other requirement**
SD	weight 0	no	none
CV	weight 0	no	non-zero mean (positive for non-negative weights)
EWM (entropy)	weight 0	non-negative	$0 \cdot \ln 0$ for zeros
CRITIC	undefined (correlation)	no	remove constant criteria
MEREC	weight 0	yes	none
LOPCOW	undefined (division by zero)	no	remove constant criteria
CILOS	undefined / ill-conditioned	yes	remove constant criteria

Source: author's analysis (Proposition 4).

7. A Worked Illustration

The following generated 5×4 benefit matrix makes the results of Sections 5 and 6 concrete. The numbers are invented and chosen so that criteria 1 and 2 are almost perfectly correlated, criteria 3 and 4 are strongly correlated and criterion 4 is nearly constant:

$$X = \begin{bmatrix} 10 & 20 & 5 & 50 \\ 20 & 38 & 9 & 52 \\ 30 & 61 & 2 & 48 \\ 40 & 79 & 8 & 51 \\ 50 & 100 & 4 & 49 \end{bmatrix}.$$

The linear correlations are $\rho_{12} = 0.999$, $\rho_{34} = 0.988$ and the cross-pair correlations are negative (near -0.3).

Table 6 reports the weights produced by each method under sum normalisation (with *CV*, *MEREC* and *CILOS* using their own scale-free constructions). Three features stand out. First, criterion 4, the near-constant one, receives the smallest weight from *SD*, *CV*, the entropy weight method and *CRITIC*, but the largest weight from *LOPCOW* and from *CILOS*. The methods can reflect different, and in this example opposite, definitions of importance: *SD*, *CV* and entropy reward dispersion, so a flat criterion is unimportant; *LOPCOW* rewards a small standard deviation relative to the level; and *CILOS* rewards small impact loss, which a flat criterion also exhibits. Second, although criteria 1 and 2 are nearly redundant, *CRITIC* does not down-weight them here, because each conflicts strongly with criteria 3 and 4; *CRITIC*'s conflict term rewards that cross-pair conflict and offsets the within-pair redundancy. Third, *IDOCRIW*, the product of entropy and *CILOS* weights, stays close to the entropy weights because the near-zero entropy weight of criterion 4 suppresses the large *CILOS* weight there.

Table 6. Weights under sum normalisation (criterion order C1, C2, C3, C4).

Method	**C1**	**C2**	**C3**	**C4**
SD	0.328	0.332	0.320	0.020
CV	0.328	0.332	0.320	0.020
EWM (entropy)	0.337	0.345	0.317	0.001
CRITIC	0.328	0.342	0.309	0.021
MEREC	0.328	0.324	0.334	0.014
LOPCOW	0.139	0.137	0.142	0.581
CILOS	0.074	0.072	0.080	0.773
IDOCRIW	0.329	0.326	0.334	0.011

Source: author's computations on the generated matrix. *SD* ***and*** *CV* ***coincide, as Proposition 2 predicts.***

Table 7 isolates the invariance result by recomputing two methods under the three pure-scaling normalisations. The entropy weights are identical to three decimal places across sum, vector and max, exactly as Proposition 1 states; the *SD* weights drift from one normalisation to the next, so an analyst who reports *SD* weights without naming the normalisation has under-specified the calculation.

Table 7. Entropy weights are invariant across pure-scaling normalisations; SD weights are not.

Normalisation	*SD* (C1-C4)	*EWM* (C1-C4)
sum	0.328 / 0.332 / 0.320 / 0.020	0.337 / 0.345 / 0.317 / 0.001
vector	0.327 / 0.330 / 0.321 / 0.022	0.337 / 0.345 / 0.317 / 0.001
max	0.321 / 0.323 / 0.325 / 0.031	0.337 / 0.345 / 0.317 / 0.001

Source: author's computations. Adding a constant fifth criterion leaves SD and entropy well-defined (weight zero) but returns an undefined vector for LOPCOW and CILOS, illustrating Proposition 4.

8. A Property-Based Taxonomy

The two results, together with the decomposition, suggest a taxonomy along three axes that are more informative than the usual subjective/objective split. The first axis is the *information source*: internal methods (*SD*, *CV*, the entropy weight method, *LOPCOW*) read only each criterion's own dispersion, whereas structural methods (*CRITIC*, *MEREC*, *CILOS*) also use information beyond each criterion's own spread: *CRITIC* through inter-criterion correlation, *MEREC* through each criterion's marginal effect on the aggregate score and *CILOS* through mutual impact losses. The second axis is the *invariance class* of Section 5: scale-invariant methods (*CV*, entropy, *LOPCOW*, *MEREC*, *CILOS*) versus normalisation-sensitive methods (*SD*, *CRITIC*). The third axis is *data admissibility* from Section 6. Table 8 places the catalogue on these axes.

Table 8. Property-based taxonomy.

Method	Information source	Invariance class	Data admissibility
SD	internal dispersion	normalisation-sensitive	any real data
CV	internal dispersion	scale-invariant	non-zero mean (positive for non-negative weights)
EWM (entropy)	internal information	scale-invariant	non-negative; $0 \cdot \ln 0$ for zeros
CRITIC	dispersion + conflict	normalisation-sensitive	no constant criteria
MEREC	structural removal effect	scale-invariant	positive
LOPCOW	internal dispersion (inverse)	scale-invariant	no constant criteria (nonzero *SD*)
CILOS	structural impact loss	scale-invariant	positive; no constant criteria

Source: author's synthesis from Sections 3, 5 and 6.

9. Reporting and Selection Protocol

The framework converts into a short protocol for applied studies that use an objective weighting method, summarised in Table 9.

First, *report the normalisation together with the method*. For *SD* and *CRITIC* the weight vector is not determined by the method alone; swapping the normalisation changes the weights, as Table 7 shows. A study that names only the method may leave the weight calculation underspecified. Second, *screen the decision matrix before weighting*: remove or merge constant criteria and check for negative entries, zero column sums and zero denominators. *CRITIC*, *LOPCOW* and *CILOS* are not well-defined on a constant criterion. The entropy weight method requires non-negative entries and a positive column sum, with the convention $0 \ln 0 = 0$ for zero entries; *MEREC* requires positive entries; and *CILOS* requires positive impact-loss denominators. Third, *match the method to the structure of the problem*: if inter-criterion conflict or redundancy is central, prefer a method that explicitly models relations among criteria, such as *CRITIC* or *CILOS*; if instead the marginal contribution of a criterion to an aggregate score is the focus, *MEREC* is more directly aligned with that interpretation; purely internal methods do not explicitly discount correlated criteria. Fourth, *be explicit about the notion of importance*: *SD*, *CV* and the entropy weight method reward dispersion, whereas *LOPCOW* emphasises low dispersion relative to level and *CILOS* can give high weight to criteria with small impact losses, which can invert the resulting order, as Table 6 demonstrates. Fifth, *interpret comparisons of near-equivalent methods cautiously*: by Proposition 2, *SD* and *CV* coincide under sum normalisation, so comparing them there does not provide an independent comparison.

Table 9. Reporting and selection guide.

Situation	**Recommended action**
Any objective weighting	report the normalisation with the method
Constant / near-constant criterion present	screen it out; avoid *CRITIC*, *LOPCOW*, *CILOS*
Negative entries, zero column sums or zero denominators present	repair the data first; entropy allows zeros only with $0 \ln 0 = 0$, *MEREC* requires positive entries, *CILOS* requires positive denominators
Conflict or redundancy among criteria is central	prefer a method that models criterion relations (*CRITIC*, *CILOS*)
Marginal contribution to an aggregate score is the focus	*MEREC*

Situation	Recommended action
Normalisation should not affect weights	prefer *EWM*, *CV*, *LOPCOW*, *MEREC* or *CILOS*
Dispersion should mean importance	prefer *SD*, *CV* or *EWM* (not *LOPCOW* or *CILOS*)

Source: author's distillation of Sections 5, 6 and 8.

10. Conclusion

Objective weighting is often treated as a fixed preliminary step, with the method chosen by convention and the normalisation left implicit. This chapter has argued that the choice is consequential and analysable. A single decomposition (normalisation, dispersion functional, structural factor) places the per-criterion methods in one comparable form and locates *CILOS* and *IDOCRIW* as the system-based structural class at its boundary. Within it, the normalisation-invariance result classifies which methods are unaffected by the pure-scaling normalisations considered here and which are not, and the degeneracy result identifies the inputs on which each method returns a zero weight, an undefined weight or fails to be well-defined. A generated example illustrates both and exposes a further contrast: the methods can reflect different definitions of what makes a criterion important, so that a near-constant criterion can be ranked least or most important depending on the method. The contribution is not a new weighting method but a common analytical frame that clarifies when the selected objective methods agree, when they differ and when they become undefined. The resulting taxonomy and reporting protocol require no new computation and can be applied before any data are collected, and they delimit what a comparison of objective weighting methods can establish.

Declarations

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