Current Approaches in Applied Statistics - I

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Preface

This book, titled Current Approaches in Applied Statistics, is a compilation of recent academic studies produced by researchers from different disciplines. The book covers not only theoretical contributions in the field of statistics, but also the innovative dimensions of methods used in a wide variety of application areas.

Today's rapidly increasing volume and diversity of data has led statistics to transcend being a science based solely on mathematical foundations and take on a critical role in many fields, from health sciences to engineering, social sciences to environmental research. Reflecting this broad sphere of influence, this book aims to present readers with both theoretical approaches and application examples from different disciplines.

The chapters in this book, prepared with contributions from international researchers, highlight the current importance of statistics, the methodological challenges encountered, and new solutions. Readers will encounter content that is useful both academically and practically in areas such as statistical modeling, data mining, machine learning, biostatistics, and social statistics.

We believe this work will provide researchers, graduate students, and practitioners with a comprehensive overview of current approaches to statistics. We thank all the authors and reviewers who contributed to this book and hope it will make a valuable contribution to the scientific community.

Contents

Chapter 1

Testing the Relative Purchasing Power Parity in Türkiye: Comparing the Headline and Core Inflation

Ahmet Arvas[1]

Mercan Hatipoğlu[2]

Abstract

After the officially abandon of the Bretton Woods Agreements in the 1973, flexible foreign rates were adopted and exchange markets were allowed fluctuate freely. Thus, the question of how to resolve the value of exchange rates came to the agenda. The first theory that came to mind was Purchasing Power Parity. The notion of PPP embraces the idea that flexible foreign exchange rates adjust themselves right away according to inflation rates. Therefore, PPP asserts that the currency of the higher inflation country should be depreciated by the inflation difference. This paper examines the validity of the relative purchasing power parity (PPP) for Türkiye with its major trading partners: the USA, the UK and Euro area. To do so, simple linear regression models is employed to quarterly data over the period 2002–2023. The empirical findings illustrate that PPP is invalid for major partner currencies ($, £, €) since exchange rate movements and inflation differentials are not identical. However, when the headline inflation is used instead of core inflation, findings show that variations in exchange rates become more tied to inflation rates. Nevertheless, results also emphasize that Turkish Lira deprecation can be attributed other factors than inflation.

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INTRODUCTION

The price of the exchange rate gives investors an idea about many issues. For example, the performance of economy, whether the currency is accepted in world trade or the country's financial risk are first topics that come to mind (Nyambuu and Tapiero, 2018: 197). One of the variables most closely related to the exchange rate is inflation. When inflation starts to rise in a country, its impact on exchange rates is immediately felt. Firstly, the country's export demand decreases due to higher prices, which reduces the demand for the local currency. Secondly, since foreign goods will become relatively cheaper, the demand for foreign currency increases. Respectively, the reduced demand for local goods and increased demand for foreign goods simultaneously place downward pressure on the value of local currency and causing the depreciation of exchange rate (Madura and Fox, 2023: 265).

The first theory that relates exchange markets and consumer inflation is purchasing power parity (PPP). Simply put, PPP defined by Gustav Cassel, is an empirical proposition which defends the movements of exchange rates are driven by the inflation differential of two countries. PPP emerged because of how exchange rates should be determined after World War I. Because the value of pre-war exchange rates was determined by the amount of gold reserves that countries had. However, after the war, the possibility of governments attempting to rebuild their countries by printing money made it impossible to maintain the gold standard (Rogoff, 1996). PPP has two types of applications. One is Absolute PPP which defends that the exchange rate should be equal to the ratio of price levels. The other is Relative PPP which claims the inflation differential across the countries is underlying factor to determine the currency rate (Solnik and McLeavey, 2014: 83).

Many authors have listed several leading factor why purchasing power parity is invalid. To summarize, Rogoff (2007) points the volatility differential of price indices and exchange rates as causes of deviations from PPP. According to his findings, variations in commodity prices are markedly smaller than exchange rates. In another study, Rogoff (1996) suggested that value-added taxes also lead to deviation. Yoon and Jei (2019) cited downward rigidity of wage as the reason why the PPP is invalid. According to Solnik and McLeavey (2014:65, 78) rents, labor costs and sticky in good prices causes the departures from PPP especially in the short run. Similarly, Miles and Scott (2008:500) argued that transportation costs, border effects and market pricing invalidate the law of one price. Melvin and Norrbin (2017:135) noted that the consumers live in different countries choose different of basket goods. Thus, preparing the price index of each country with different weighted consumption patterns and nontraded goods causes deviations to occur. Despite shortcomings, the reason why PPP is still worth researching is that foreign exchange rates sooner or later revert to their fundamental values (Solnik and McLeavey, 2014: 90).

This study investigates validity of Relative PPP hypothesis for Türkiye from 2002:Q1 to 2023:Q4 using OLS regression. This paper differs from existing literature on two fronts. While headline inflation rates are used in many studies (Telatar and Kazdagli, 1998; Doganlar,1999; Yazgan, 2003; Alba and Park, 2005; Yıldırım, 2017; Özmen and Gökcan, 2004;) on Türkiye to determine purchasing power parity, this study use non-food, non-energy CPI. In addition, the general tendency in literature (Koncak and Güriş (2022), Coşkun (2020), Doğanlar et al. (2020), Doğanlar et al.(2021), Erdoğmuş (2021), Uğur and Alper (2023)) conducted for Türkiye is to test absolute PPP based on movements of the real exchange rate.

Apart from existing literature, this study tests Relative PPP by taking into account the inflation differences between the two countries. The parts of the study are classified in this fashion. Second section includes the literature review. Section 3 is devoted to overview of monetary policy and inflation in Türkiye. Fourth section expresses the data with model. Section 5 discusses the empirical findings. Finally, last part is the conclusion and contains some policy recommendation.

LITERATURE REVIEW

Considering that purchasing power parity has such a long history, it is not surprising that the literature is full of mixed findings. While early studies of Gailliot (1970), Officer (1978), Rush and Husted (1985) and Kim (1990) empirically supported the PPP hypothesis in general, conversely papers of Adler and Lehmann (1983), Abuaf and Jorion (1990) and Patel (1990) did not support for long run PPP. When we look at studies conducted with cointegration techniques, Varamini and Lisachuk (1998), Salehizadeh and Taylor (1999) and Arize, Malindretos and Nam confirmed the validation of PPP respectively for Ukraine emerging countries and Africa countries. However, the studies of Chocholatá (2009) and Baharumshah and Ariff (1997), Jacobo, and Sosvilla-Rivero (2021) that failed to backing purchasing power parity in cases of Slovakia, Asian countries and Argentina respectively, applying similar techniques. The studies of Zhou (1997), Haug and Basher (2011) have given partial support to the PPP hypothesis and stated that PPP is valid in the weak form. Again, many studies drawing attention to the nonlinear nexus exchange rates and have tested PPP using nonlinear time series models. For example, Bozoklu and Kutlu (2012), Su and Chang (2011), Baum et al. (2001) and the findings of Su et al.(2002) encourages purchasing power parity. However, using similar techniques, Tiwari and Shahbaz, (2014) rejected the PPP hypothesis for India. Studies conducted for euro area, OECD, ASEAN-5, G7 and 159 international countries respectively by Koedijk et al. (2004), Kalyoncu and Kalyoncu (2008), Munir and Kok (2015), Kargbo, (2009) and Vo and Vo (2023) found support for PPP. Considering the studies conducted based on Tukiye, it is clearly seen that PPP is a controversial issue because many paper presented mixed findings. Koncak and Güriş (2022), Doğanlar et al.(2020), Doğanlar et al.(2021) and Uğur and Alper (2023) confirmed PPP's validity for Türkiye, whereas Azazi et al.(2023), Erdoğmuş (2021) and Coşkun (2020) concluded that PPP is not valid.

OVERVIEW OF MONETARY POLICY AND INFLATION IN TÜRKİYE

Türkiye allowed the exchange rates to float freely after 2001 crisis and again as of the same year The Central Bank of Türkiye adopted inflation targeting model. While the fixed exchange rate regime was applied in Türkiye until 1980, roughly crawling peg exchange rate and managed float regime was adopted between the episodes 1980-2001 (Leigh and Rossi, 2002). Particularly, between 1980 and 1989, the aim of exchange rate policy was to support export-led growth strategy. In later periods, the nominal value of the Turkish Lira was depreciated by the central bank in line with inflation expectations (Kandil et al., 2007).

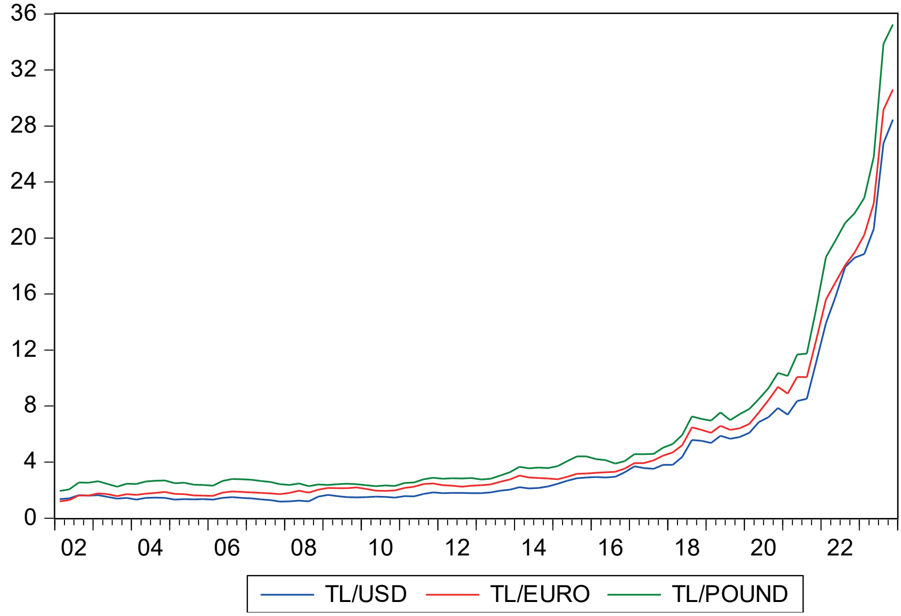

Figure 1. Value of TL against major currencies

Source: CBRT Department of Statistics

Overall, figure 1 shows that after remaining stable, the TL depreciated strongly relative to other currencies since the mid of 2019. During the 2002-2012 episodes, no abrupt movement was observed in exchange rates due to price stability. As of 2023, it seems that major currencies reached all-time high record against TL.

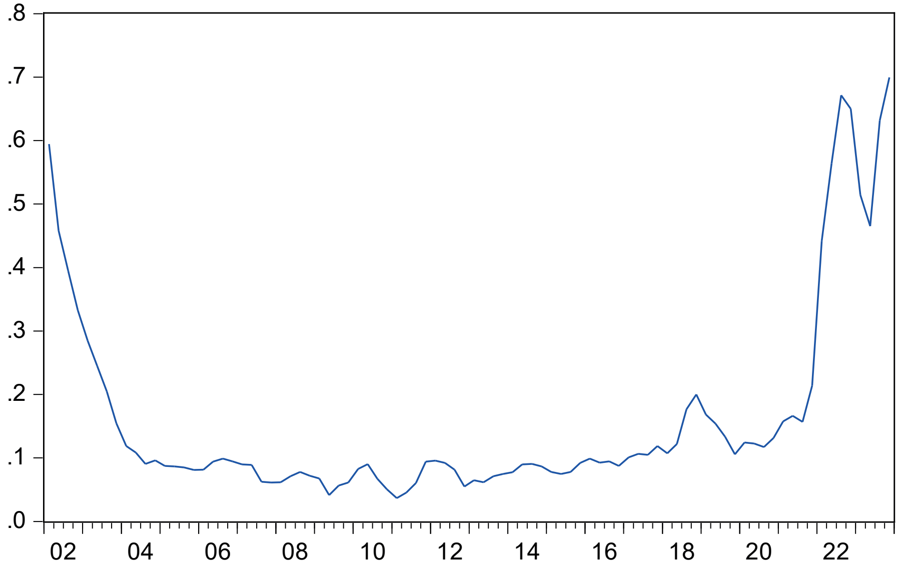

Figure 2.Türkiye inflation rate between 2002Q1-2023Q4, %

Source: CBRT Department of Statistics

Türkiye has suffered from stubborn inflation for years. Inflation rates reached very high levels in 2002 and 2021. In fact, inflation declined markedly from 60 percent in 2002 to 10 percent in 2004 within two years and before climbing back to 20 percent in 2021, stayed moderate levels from 2004 to 2012. The inflation targeting regime worked reasonably well during the period 2004-2012. Nevertheless, covering the period 2002-2023, inflation averaged about %16 quarterly, meaning that price stability has not yet been achieved. According to Cecchetti and Schoenholtz (2017), the causes of the inflation may be attributed highly to mistakes in monetary policy. However, the failed fiscal policy as a result of politicians spending more than necessary by relying on the central bank's ability to print money is also shown another reason. In some articles from the Turkish economic literature, the causes of inflation are attributed to other issue. For example, Demiralp and Demiralp (2019) stated that declining central bank of Türkiye independence result in weakening inflation targeting regime. In similar vein, Gürkaynak et al., (2023) emphasized that the central bank's early interest rate cut decisions are the main reason for the recent jump in inflation. Lastly, Yilmazkuday (2022) concluded by using VAR models that economic drivers of Turkish inflation are oil price fluctuation and US dollar rate.

DATA AND EMPIRICAL MODEL

Data

The dataset consists of quarterly observations which covers the period from 2002 Q1 to 2023:Q4 (totally 88 quarters) for Türkiye and its three major partners Euro Area, the UK, and the USA. The reason why it was started in 2002 is that the value of the Turkish Lira has been determined by market forces since that year. The inflation data set includes both headline consumer inflation and Core inflation (non-food non-energy consumer inflation) is obtained from the OECD Statistics database. The nominal exchange rates are derived from data system of The Central Bank of Türkiye.

Econometric Methodology

The paper benefited from the following ordinary least squares (OLS) regression model similar to the methods used in studies of Mishkin (1984) and Nyambuu and Tapiero (2018):

$$\%\Delta S_t = \alpha + \beta\left(\pi_t^{TUR} + \pi_t^{F}\right) + \varepsilon_t \qquad (1)$$

Where $\%\Delta S_t$ is a percentage change in spot exchange; π^{TUR} and π^{F} are percentage changes in inflation of Türkiye and foreign countries, respectively and finally ε_t is a residual term distributed with constant variance, uncorrelated with one another and expected mean value is zero. Whereas the α parameter stands for intercept and is the expected deprecation or appreciation of currency, the β parameter demonstrates the effect of inflation differentials on currencies. If the relative version of PPP holds, β must be equal to 1.

FINDINGS

Summary statistics

Table 1 illustrates the descriptive statistics of the bilateral of inflation differentials and exchange rates between Türkiye and the USA, Euro zone and the UK. The p-values of the Jarque–Bera (JB) tests clearly indicate that all series have non-normal distribution. Both the means (expected value) and standard deviations (risk) of exchange rates are very close to each other. Because international foreign exchange markets do not allow arbitrage, when TL depreciates, it loses equally value against all currencies. The same is almost valid for inflation differences. Whereas the means of inflation differences ranges between 0.13 to 0.14, standard deviations varies between 0.15 and 0.16. However, it should be emphasized that core inflation differentials fluctuate relatively higher than headline inflation. A glance at table reports positive skewness for all of inflation differentials and exchange rates. In terms of exchange rates, the findings denote the depreciations of TL outweigh the appreciations against major currencies. The kurtosis values of inflation differentials supports the the prevalence of fat tail distributions with more outliers. This fact is a result of sudden jumps in inflation in Türkiye.

Table 1.Summary statistics of inflation differentials and exchange rates

	TR-USA			TR-EURO			TR-UK		
	ID^h	ID^c	%ΔS	ID^h	ID^c	%ΔS	ID^h	ID^c	%ΔS
Mean	0.13	0.14	0.03	0.14	0.14	0.03	0.14	0.14	0.03
Std. Dev	0.15	0.16	0.07	0.15	0.16	0.07	0.15	0.16	0.07
Skewness	2.11	2.24	1.02	2.09	2.22	0.89	2.06	2.21	0.98
Kurtosis	6.33	7.02	4.17	6.29	6.97	3.97	6.13	6.96	4.23
JB [Prob]	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Correlation*	0.33	0.34		0.37	0.36		0.35	0.33	

Source: Author's computation

ID^h: headline inflation differential, ID^c: headline inflation differential, %ΔS: spot exchange rate return

**** Correlation with exchange rate***

Unit Root Results

Table 2. Phillips-Perron unit root tests results

	Level			First difference of log level		
	constant	constant with trend	None	constant	constant with trend	None
TL/USD	22.93	17.25	10.11	-7.28***	-8.95***	-6.39***
TL/EURO	22.90	37.84	9.92	-7.62***	-8.42***	-6.46***
TL/POUND	29.89	24.03	11.11	-7.39***	-8.33***	-6.47***
Headline						
$\pi^{TR} - \pi^{USA}$	-1.22	-2.01	-0.77	-5.45***	-5.91***	-5.51***
$\pi^{TR} - \pi^{EURO}$	-1.23	-2.13	-0.75	-5.41***	-5.89***	-5.47***
$\pi^{TR} - \pi^{UK}$	-1.36	-2.17	-0.90	-5.42***	-5.92***	-5.48***
Core						
$\pi^{TR} - \pi^{USA}$	-2.70*	-3.58**	-2.03**	-6.27***	-7.01***	-6.24***
$\pi^{TR} - \pi^{EURO}$	-2.54*	-3.53**	-1.74**	-6.14***	-6.87***	-6.16***

$\pi^{TR} - \pi^{UK}$	-2.65*	-3.55**	-1.90**	-6.18***	-6.94***	-6.20***
critical values						
1% level	-3.50	-4.06	-2.59	-3.50	-4.06	-2.59
5% level	-2.89	-3.46	-1.94	-2.89	-3.46	-1.94
10% level	-2.58	-3.15	-1.61	-2.85	-3.15	-1.61

Source: Author's computation

For checking the stationary of variables, the Phillips-Perron (1988) unit root test was preferred constant, constant with trend and as well as none. Table 2 revealed that all series appear to be non-stationary in level forms except the core inflation differentials. However, when log-first differences of each variable were used, non- stationarity was rejected in all the cases.

Regression results

Table 3. OLS Regression results of relative PPP

	With Headline inflation			With Core Inflation		
	USA	EURO	UK	USA	EURO	UK
α	0.03***	0.03***	0.03***	0.03***	0.03***	0.03***
β	0.79***	0.50**	0.56**	0.50***	0.29**	0.32*
R^2	0.21	0.10	0.12	0.15	0.05	0.07
LM [1]	0.29	0.11	0.07	0.15	0.08	0.04
ARCH [1]	0.87	0.47	0.82	0.71	036	0.49
F-Stat.	23.90	10.27	12.29	0.13	5.33	6.47
F-Prob.	0.00	0.00	0.00	0.00	0.00	0.01

Source: Author's computation

Significant at: 1%; 5%**; 10%****

Without exception, table 3 demonstrates positive β coefficients that are less than 1. In other words, econometric results are consistent with idea that inflation differentials have a statistically significant influence on exchange rates. Since beta coefficients are in the range of 0.32 to 0.76, table presents little empirical evidence in favor of PPP. Turning to the β coefficients in table, one percent increase in headline inflation differentials leads to depreciates TL by 0.79, 0.50 and 0.56 percent, against the USA dollar, the Euro and the pound, respectively. On the other hand, taking core inflation rate into consideration, the effect of inflation differences on the exchange rate weakens significantly. In the case of core inflation, a one percent expansion between inflation differences causes the Turkish lira to lose value by 0.50, 0.29 and 0.32 percent, against the currencies of USA, Eurozone, and UK, respectively. Overall, findings indicate that PPP does not valid because the exchange rate movements and inflation differentials are not identical. The facts that the coefficients are not equal to 1 confirm the inflation differences are not fully reflected in the exchange rate. In addition, the headline inflation data offer more support for relative PPP than core inflation.

Of course, the failure of validation of PPP does not imply that the exchange rate cannot achieve in reflecting inflation differentials. For example if Turkish inflation is higher than U.S. inflation, the Turkish lira will depreciate relative to the dollar but the depreciation is not be one-to-one, but will be half of the inflation differences. For this reason, changes in exchange rates tied to differences in inflation rates, even if there is a weak linkage.

In terms of R^2 , all three countries have low value, indicating that models fit the data poorly. The R^2 of models vary from the 0.05 to 0.21, implying that inflation differentials with optimistic forecast, explain less than 20% in the variations of exchange rate movements. This means that the remaining 80% of foreign exchange movements can be attributed factors other than inflation. Finally, table 3 shows that both LM and ARCH probability values indicate no any remaining serial correlation and heteroscedasticity of the error term.

CONCLUSION

Turkish Lira is free floating and fully determined by non-government market forces that reflect economic performance and future expectations at each point in time. Also, Lira have been fluctuating according to supply and demand freely especially after 2002. The findings of the study indicate that the price of exchange rate is not as simple as presumed by PPP. Nevertheless, it is worth stressing that TL will depreciate against major currencies quite markedly depending on the type of price index. Therefore the currency of Türkiye with higher inflation tends to weaken over time against the US dollar, the Euro and the British pound. Moreover, since the depreciation of the TL is not offset by the inflation difference precisely, Türkiye will be less competitive in the international market. To sum up, widening inflation differential feeds the depreciation of the TL. The monetary authorities should have to close the gap by implementing tightening monetary policy.

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Chapter 2

A Computational Study on Sobol' Sequences

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Abstract

This study presents a computational comparison between Quasi-Monte Carlo (QMC) methods based on Sobol' sequences and traditional Monte Carlo (MC) methods using the Mersenne Twister (MT) generator. While Sobol' sequences are widely recognized for outperforming MT in terms of convergence, our results reveal notable deficiencies when applied to high-dimensional Geometric Asian option pricing. To investigate this behavior, we conduct moment and correlation analyses, identifying a bias in the incremental construction of Sobol' paths—a bias that is absent in MT and can be alleviated through skipping initial points, scrambling, or Brownian Bridge (BB) techniques. All simulations are implemented in Python, with additional acceleration achieved through Graphics Processing Unit (GPU)-based parallel computing environments.

INTRODUCTION

The motivation for this work stems from the complexities of pricing exotic derivatives under models demanding numerous time steps, thereby creating highly dimensional Brownian motion trajectories. In financial engineering, it is often impossible to derive closed-form solutions for the valuation of financial products, especially those categorized as exotic options. As a result, numerical techniques—and Monte Carlo (MC) simulation methods in particular—play a critical role. As discussed extensively by Glasserman (Glasserman, 2014), the appeal of Monte Carlo methods lies in their general applicability, especially in cases where analytical solutions are infeasible or unavailable. In MC simulations, the proper use of random number

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generators is vital, as incorrectly generated random sequences can lead to severe mispricing of financial products, potentially resulting in significant financial losses for institutions and individuals involved in their trading.

Traditional Monte Carlo method relies on pseudo-random number generators (PRNGs), such as the widely used Mersenne Twister, to simulate stochastic processes. However, the convergence rate of classical MC is typically $\mathcal{O}\left(N^{-1/2}\right)$, where N is the number of samples. This achieving high accuracy requires generating millions of paths, which can be computationally intensive and slow to converge (Glasserman, 2014 and Jäckel, 2002).

Quasi-Monte Carlo (QMC) methods aim to improve upon this by replacing pseudo-random numbers with deterministic low-discrepancy sequences, such as Sobol' or Halton sequences. These sequences are designed to fill the simulation space more evenly, reducing clustering and improving convergence to approximately $\mathcal{O}\left(\frac{(\log N)^d}{N}\right)$ as stated in (Jäckel, 2002). In particular, Sobol' sequences have gained significant attention due to their simplicity, extensibility in high dimensions, and well-understood mathematical properties.

Despite their theoretical advantages, QMC methods are sensitive to implementation details. As noted by Glasserman (Glasserman, 2004), implementation choices such as scrambling—to improve uniformity and mitigate artifacts—and the ordering of dimensions can significantly influence the accuracy and stability of QMC results. Additionally, techniques like Brownian bridge (BB) construction are often employed in QMC to reorder variance allocation in time-dependent simulations, thereby improving efficiency for path-dependent derivatives.

This study provides a practical and empirical evaluation of Sobol' sequences in high-dimensional Monte Carlo simulations. We benchmark the performance of Sobol implementation against the Mersenne Twister, across a set of controlled experiments that include:

- One- and multi-dimensional integration tests
- Geometric Asian option pricing under a geometric Brownian motion model
- Moment and correlation analysis among Sobol' dimensions
- Effects of scrambling, skipping initial points, and Brownian bridge
- Graphics Processing Unit (GPU)-accelerated simulations

Our main objective is to determine under what conditions QMC methods, particularly Sobol' sequences, truly outperform traditional pseudo-random simulations in practice, and to investigate the performance gains offered by GPU-based parallel computation in high-dimensional Monte Carlo simulations.

BACKGROUND AND RELATED WORK

Monte Carlo Methods

Monte Carlo methods are widely used to approximate expectations by drawing random samples from a given probability distribution (Glasserman, 2004). These methods operate by repeatedly sampling and evaluating a function of interest, thereby estimating integrals or probabilities numerically. As the number of samples increases, the estimate converges to the true value by virtue of the law of large numbers. Furthermore, the central limit theorem enables quantification of the uncertainty in the estimate by providing asymptotic confidence intervals and standard errors.

To illustrate the method, let us use the example from Section 1.1.2 of (Glasserman, 2004). Consider the pricing of a European call option using the Black Scholes framework. The payoff of the option depends on the terminal value of the underlying stock, which follows a lognormal distribution as a result of modeling the asset price dynamics with a Geometric Brownian Motion (GBM). By generating standard normal samples, we can simulate the terminal stock price and compute the discounted payoff. The process is summarized in Algorithm 1 (see Figure 1), which shows the basic steps for estimating the expected present value of the option using Monte Carlo simulation. In this algorithm, r denotes the risk-free interest rate, ó is the volatility of the underlying asset, S_0 is the initial asset price, K is the strike price, T is the time to maturity, and $Z_i \sim \mathcal{N}(0,1)$ represents independent standard normal variables.

Algorithm 1 Monte Carlo estimation of expected present value of a European call option

for $i \leftarrow 1$ to n **do**
    Generate $Z_i \sim \mathcal{N}(0,1)$
    $S_i(T) \leftarrow S_0 \cdot \exp\left(\left(r - \frac{1}{2}\sigma^2\right) T + \sigma\sqrt{T} \cdot Z_i\right)$
    $C_i \leftarrow e^{-rT} \cdot \max(S_i(T) - K, 0)$
end for
Estimate: $\hat{C}_n = \frac{1}{n} \sum_{i=1}^{n} C_i$

Figure 1. Monte Carlo estimation of a European call option

In classical MC simulation, PRNGs such as the widely used Mersenne Twister are employed to sample independent and identically distributed random variables from target distributions. Although these methods exhibit a convergence rate of $\mathcal{O}\left(N^{-1/2}\right)$, their efficiency diminishes in problems that require high precision or suffer from the curse of dimensionality. See, for instance (Glasserman, 2004 and Jäckel, 2002).

To address these limitations, QMC methods utilize *low-discrepancy sequences (LDS)* to fill the integration domain more uniformly than pseudo-random samples. This deterministic structure can improve the convergence rate to $\mathcal{O}\left(\frac{(\log N)^d}{N}\right)$ for d-dimensional integrals under sufficient smoothness conditions (Glasserman, 2004, Jäckel, 2002 and Sobol' and Kucherenko, 2005). The discrepancy of a point set intuitively measures its deviation from uniformity, where lower discrepancy yields better space-filling.

Pseudo-Random Generators: Mersenne Twister

The Mersenne Twister is one of the most commonly used PRNGs due to its long period $2^{19937}-1$, fast generation speed, and guaranteed equidistribution in at least 623 dimensions (Jäckel, 2002). While its output is not truly random, it produces sequences that behave uncorrelated and independent for practical purposes, making it suitable for many high-dimensional Monte Carlo simulations.

In comparative studies, MT serves as a baseline for measuring QMC effectiveness, particularly in terms of convergence rate, variance, and dimensional correlation.

Low-Discrepancy Sequences: Sobol'

One of the most widely adopted low-discrepancy sequences in QMC methods is the Sobol' sequence, originally introduced in (Sobol', 1967) by I. M. Sobol' in 1967 and further developed in 1976. See (Sobol', 1976). These sequences are constructed using *direction numbers* and *Gray code* ordering to ensure uniform coverage over the unit hypercube $[0,1)^d$. Due to their simplicity of construction, scalability to high dimensions, and successful applications in finance, Sobol' sequences have become a standard choice in QMC simulations (Jäckel, 2002).

Despite their advantages, the effectiveness of Sobol' sequences in high-dimensional settings is not always guaranteed. High-dimensional implementations, such as BRODA's Sobol65536 generator as documented in (Broda, 2025), aim to scale QMC techniques to tens of thousands of dimensions. However, empirical studies (Sobol', 1967 and Silva and Barbe, 2005) have observed issues such as dimensional correlation and loss of uniformity, which can negatively impact simulation accuracy. This is especially problematic in financial applications, where small deviations in distribution quality may lead to significant pricing and risk estimation errors.

Several practical enhancements have been proposed to improve the robustness of Sobol' sequences. Among these, three key factors are frequently highlighted:

- The use of *scrambling*, which introduces controlled randomness to eliminate structural artifacts and enable error estimation (Owen, 1998)
- Proper *dimensional ordering*, which assigns the most influential variables (if known) to the earliest dimensions, where Sobol' sequences typically exhibit better uniformity properties (Silva and Barbe, 2005)
- Careful treatment of the *initial points*, especially the very first point (typically $(0,0,\ldots,0)$ (Owen, 2021)

Scrambling was first introduced by Owen (Owen, 1998) to address the deterministic structure of low-discrepancy sequences, which limits error estimation through confidence intervals. In scrambling, the digits of the b-ary expansion of each point are randomly permuted in a recursive and structured manner. At the j-th digit, there are b^{j-1} partitions, each independently shuffled. This technique improves space-filling properties while retaining the low-discrepancy nature of the original sequence.

Another discussed topic in literature is the practice of skipping initial Sobol' points (also known as fast-forwarding), which involves discarding the first few values of the sequence—typically starting from the all-zero point $(0,0,\ldots,0)$. This is often motivated by the observation that early Sobol' points can lead to undesirable properties, particularly when passed through nonlinear transformations such as the inverse normal CDF Ö^{-1}, to produce normally distributed variates. Under certain conditions, skipping has been shown to reduce numerical integration error, albeit without changing its asymptotic order (Radovic et al., 1996). However, the optimal number of points to skip is problem-dependent and cannot easily be determined. Moreover, as Owen (Owen, 2021) notes, indiscriminate skipping—especially when combined with scrambling—may disrupt the digital net structure of the sequence and lead to worse convergence behavior. As such, the handling of initial points demands careful consideration and should ideally be validated within the specific simulation context.

A more algorithmic enhancement that complements Sobol' sequences is the Brownian bridge construction, a technique designed to improve the dimensional efficiency of path simulations. Brownian Bridge reorders the discretization of stochastic processes to concentrate variance in the early time steps. This reordering aligns well with Sobol' sequences, whose early dimensions are more uniformly distributed, allowing the most critical components of the process to benefit from the best uniformity. As a result, Brownian bridge improves both dimensional ordering and convergence properties (Glasserman, 2004, Kucherenko and Shah, 2007 and Bianchetti et al., 2015).

Given the high computational demands of large-scale QMC simulations, leveraging parallel computing architectures has become essential. In particular, GPUs offer a powerful platform for accelerating simulations due to their high degree of parallelism. Recent work by Bernemann et al (Bernemann et al., 2011) demonstrated the feasibility and effectiveness of GPU-accelerated Sobol-based simulations for pricing exotic derivatives and performing calibration tasks.

In this study, we build upon these methodological advances by evaluating GPU-based implementations of Sobol' and Mersenne Twister generators. Our focus is on understanding both the theoretical conditions and practical configurations under which Sobol' sequences, possibly enhanced with Brownian Bridge and scrambling, can outperform traditional pseudo-random approaches. The ultimate goal is to achieve high numerical accuracy while benefiting from the significant speedups offered by parallel hardware.

METHODOLOGY

This section outlines the computational setup, simulation framework, and evaluation metrics used in this study to compare Sobol-based QMC methods with classical MC methods using PRNGs, specifically the Mersenne Twister.

Tools and Computational Environment

All experiments are implemented in Python 3.12 and executed on a machine equipped with an NVIDIA GPU (CUDA enabled). The following libraries are used for simulation and numerical computation:

- *scipy*: To generate low-discrepancy Sobol' sequences, with support for scrambling and skipping.

- *numpy*: For numerical operations and random number generation using Mersenne Twister.
- *cupy*: To enable GPU-accelerated vectorized simulation paths.
- *torch*: Used specifically to generate Sobol' sequences directly on the GPU, as CuPy does not provide native support for GPU-based quasi-random sequence generation

The *Sobol* class in Scipy incorporates several important features for high-dimensional QMC simulations. It supports up to 21201 dimensions by utilizing precomputed direction numbers generated by Kuo et al. (Joe and Kuo, 2008). This capability is particularly useful in financial applications involving long time horizons, such as Asian options with daily monitoring over extended periods.

The implementation also includes a two-stage scrambling technique (Matousek, 1998) to enhance uniformity and reduce structural artifacts in the sequence. Such scrambling consists of:

- *Left Linear Matrix Scrambling (LMS)*, where direction numbers are transformed via a non-singular lower-triangular matrix to maintain low discrepancy while improving uniformity.
- *Digital Random Shift*, which applies a uniform digital shift across all dimensions to introduce randomness.

By combining these features—extended dimensional support and scrambling techniques—Scipy Sobol implementation is well-suited for high-dimensional simulations in financial engineering applications involving complex payoff structures.

Experimental Design

To provide a fair comparison between PRNG-based Monte Carlo and Sobol-based QMC, each experiment follows the same simulation logic, differing only in the source of randomness and variance reduction techniques. The following factors are controlled across experiments:

Sample size: Fixed to powers of two (e.g., 4096, $2^{19}, 2^{20}$) to suit Sobol's structure.

Dimensionality: Ranges from 1D to 21201D depending on the test, constrained by the maximum dimensionality supported by Python. In practical scenarios such as Asian option pricing, the number of dimensions corresponds to the number of discretization steps over time (e.g., 3650 for 10 years of daily steps).

Skip values: Various skip levels are tested (e.g., 1024, 2^{19}) to investigate convergence sensitivity.

Scrambling: Tests are conducted to assess the impact of randomized Sobol' sequences.

Brownian Bridge: Time steps are simulated using Brownian Bridge construction to reallocate variance into lower dimensions.

GPU usage: High-dimensional path simulations and payoff evaluations are accelerated using GPU parallelization to reduce computation time.

Evaluation Metrics

To assess simulation accuracy and efficiency, the following quantitative metrics are recorded, where $\hat{V}_i$ is the estimated value from trial i, and V_{true} is the known theoretical value (e.g., for Asian options), and N is the sample size (or, number of replications):

$$\text{Mean Absolute Error (MAE)}: = \frac{1}{N}\sum_{i=1}^{N}\left|\hat{V}_i - V_{\text{true}}\right| \quad (1)$$

$$\text{Maximum Absolute Error (MaxAE)} = \max_{1\leq i\leq N}\left|\hat{V}_i - V_{\text{true}}\right| \quad (2)$$

$$\text{Mean Error (Bias)} = \frac{1}{N}\sum_{i=1}^{N}\left(\hat{V}_i - V_{\text{true}}\right) \quad (3)$$

The elapsed time is computed using Python's time.time() function before and after simulation blocks. We remark that in high-dimensional tests, correlation matrices and variance of intermediate quantities are also recorded to study internal simulation stability. By standardizing the above setup, we ensure reproducibility and comparability across different random number sources, variance reduction techniques, and hardware acceleration strategies.

INTEGRAL TEST

To systematically evaluate the numerical accuracy of QMC and traditional MC methods, we conduct a series of one-dimensional and multi-dimensional integration experiments.

One-Dimensional Integral Test

To assess the basic numerical behavior of QMC methods in low-dimensional settings, we begin by evaluating a classical family of one-dimensional integrals of the form

$$(n+1)\int_0^1 x^n \, dx = 1, \quad n = 1, 2, \ldots, 20 \quad (4)$$

for which the analytical value is known to be exactly one for all integer values of n. This benchmark serves as a reliable testbed for quantifying integration error and convergence characteristics of different sampling methods, including pseudo-random sequences and low-discrepancy sequences such as Sobol'.

Each integral is approximated numerically using 4096 sample points. The performance of both methods is compared by computing the absolute error between the estimated and exact values across the tested range of n. For each n, the maximum and mean absolute errors are recorded in Table 1.

Table 1. Absolute errors for one-dimensional monomial integrals using Sobol and MT

n	**Sobol Max**	**Sobol Mean**	**MT Max**	**MT Mean**
1	0.00018	0.00012	0.03549	0.00716
2	0.00028	0.00018	0.05918	0.01114
3	0.00037	0.00024	0.07578	0.01416
4	0.00046	0.00031	0.08617	0.01666
5	0.00055	0.00037	0.09518	0.01884
6	0.00064	0.00043	0.10459	0.02079
7	0.00073	0.00049	0.11263	0.02256
8	0.00083	0.00055	0.11959	0.02421
9	0.00092	0.00061	0.12588	0.02574
10	0.00101	0.00067	0.13414	0.02720
11	0.00110	0.00073	0.14182	0.02858
12	0.00119	0.00079	0.14896	0.02991
13	0.00128	0.00085	0.15560	0.03117
14	0.00138	0.00092	0.16178	0.03239
15	0.00147	0.00098	0.16757	0.03357
16	0.00156	0.00104	0.17320	0.03470
17	0.00165	0.00110	0.17849	0.03580
18	0.00174	0.00116	0.18347	0.03687
19	0.00183	0.00122	0.18818	0.03791
20	0.00193	0.00128	0.19262	0.03892

The results demonstrate that Sobol' sequences yield significantly lower integration error compared to Mersenne Twister across all values of n. In particular, the maximum and mean absolute errors associated with Sobol' are consistently an order of magnitude smaller than those of MT, especially for higher-degree monomials as clearly seen in Table 1. These findings reinforce the established theoretical understanding that low-discrepancy sequences exhibit superior performance in deterministic numerical integration, especially in low-dimensional settings.

Multi-Dimensional Integral Test

To assess the accuracy and robustness of QMC methods in higher dimensions, we conducted a series of experiments based on the family of multi-dimensional integrals given in Equation 5:

$$\int_0^1 \cdots \int_0^1 \prod_{i=j}^{j+n-1} \left(1 + c_i \cdot \left(x_i - 0.5\right)\right) dx_j \cdots dx_{j+n-1} = 1 \quad (5)$$

evaluated for various values of j, n, and constant coefficients c_i The integral is designed to emphasize the role of boundary sampling: the larger the values of c_i, the more weight is placed on the integrand near the edges of the $\left[0,1\right]^n$ hypercube. This makes it an ideal stress test for comparing Sobol' sequences to MT, especially in terms of their ability to adequately capture extreme regions of the domain.

We tested several values of n ranging from 3 to 21,201 dimensions. In the small and moderate dimensional cases ($n<21201$), we employed a sliding window approach over a large Sobol' matrix of shape $\left(4096 \times 21201\right)$: for each offset j, we extracted n consecutive dimensions, computed the numerical integral, and recorded the corresponding absolute error. In the full-dimensional case ($n=21201$), a batch-based method is adopted. We performed 100 independent simulations of 4096 Sobol' samples, each preceded by a skip of $j \cdot 4096$, and reported the maximum and average absolute error across batches.

All Sobol' sequences are generated with scrambling disabled and appropriate skip values. Pseudo-random samples are initialized with batch-specific seeds to ensure independence. Each integrand is evaluated, having fixed $c_i = c$, as:

$$f\left(x\right) = \prod_{i=1}^{n} \left(1 + c \cdot \left(x_i - 0.5\right)\right) \quad (6)$$

The results are summarized in Table 2, which reports the observed maximum and mean absolute errors for both Sobol and Mersenne Twister methods across different configurations.

Table 2. Multi-dimensional integral errors for various (n,c) combinations

n and c	Sobol Max Error	Sobol Mean Error	MT Max Error	MT Mean Error
3 and 0.5	0.0157	0.00023	0.0153	0.0031
5 and 0.5	0.0192	0.00047	0.0218	0.0041
5 and 0.5 (skip 2^{19})	0.0192	0.00027	0.0218	0.0041
5 and 0.3	0.0068	0.00023	0.0132	0.0024
30 and 0.3	0.0258	0.00263	0.0334	0.0063
1000 and 0.01	0.0021	0.00073	0.0042	0.0014
21201 and 0.0002	0.000068	0.000004	0.0004	0.0001

These results clearly demonstrate the advantage of Sobol' sequences, particularly as dimensionality increases and the coefficient c decreases. In low-dimensional settings, both methods perform reasonably well, although Sobol still shows a lower mean absolute error. However, in high dimensions—especially with $n = 21201$ and $c = 0.0002$—Sobol significantly outperforms Mersenne Twister, with an error reduction factor exceeding 25 times.

An additional experiment applying a large skip of 2^{19} in the $n=5$, $c=0.5$ case also confirmed that skipping early Sobol' points leads to meaningful error reduction (from 0.00047 to 0.00027 in average error), without affecting the maximum deviation. See Table 2, thereby, we may claim that skipping can be beneficial for high-accuracy results.

Overall, these findings validate the superiority of Sobol' sequences equily well also in high-dimensional numerical integration.

GEOMETRIC ASIAN OPTION PRICING

To investigate the comparative performance of QMC methods and classical MC methods in a realistic financial context, we conducted a series of experiments pricing a geometric Asian call option under a Geometric Brownian Motion (GBM) framework: Black-Scholes setting.

The underlying asset price S_t is modeled according to the classical GBM process, defined by the stochastic differential equation:

$$dS_t = rS_t\,dt + \sigma S_t\,dW_t \quad (7)$$

where r denotes the constant risk-free rate, σ is the constant volatility of the asset, and W_t represents a standard Brownian motion. Discretization is performed using a log-Euler scheme:

$$\ln S_{t_i} = \ln S_{t_{i-1}} + \left(r - \frac{\sigma^2}{2} \right) \Delta t + \sigma \sqrt{\Delta t} \cdot z_i \quad (8)$$

where $z_i \sim \mathcal{N}(0,1)$ are independent standard normal variates generated via either pseudo-random or quasi-random sequences.

The geometric Asian call option payoff is based on the geometric mean of asset prices over discrete monitoring dates:

$$\text{Payoff} = \max(\text{GA} - K, 0), \quad \text{where} \quad \text{GA} = \left(\prod_{i=1}^{n} S_{t_i} \right)^{1/n} \quad (9)$$

with K denoting the strike price. In our experimental setup, we set $S_0 = 1$, $r = 0.05$, $\sigma = 0.4$ and $K = 1$. The maturity T corresponds to 21201 timesteps, equivalent to 10 monitoring points per day over approximately 5.8085 years:

$$t_i = \frac{i}{365 \times 10}, \quad i = 1, \ldots, 21201 \quad (10)$$

The theoretical value of the geometric Asian option is available in closed form. It is computed using an adjusted volatility and maturity:

$$\sigma_{\text{adj}} = \frac{\sigma}{\sqrt{3}}, \quad T_{\text{adj}} = \frac{T\left(r + \sigma^2 / 6\right)}{2}, \quad K_{\text{adj}} = Ke^{T_{\text{adj}}} \quad (11)$$

yielding the option price:

$$\text{Price} = e^{-T_{\text{adj}}} \cdot \text{BlackScholes}\left(S_0, K_{\text{adj}}, r, \sigma_{\text{adj}}, T_{\text{adj}}\right) \quad (12)$$

where the Black-Scholes pricing formula for European call options (Black and Scholes, 1973) is employed:

$$\text{BlackScholes}(S_0, K, r, \sigma, T) = S_0 \Phi(d_1) - Ke^{-rT} \Phi(d_2), \quad (13)$$

$$d_1 = \frac{\ln(S_0 / K) + \left(r + \frac{1}{2}\sigma^2 \right) T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

Here, $\Phi(\cdot)$ denotes the cumulative distribution function (CDF) of the standard normal distribution.

For the option under consideration the theoretical formula returns 0.19715. Simulations are conducted using two approaches: Sobol' sequences, and Mersenne Twister based pseudo-random numbers. Each simulation consists of 16 independent trials, each using 4096 paths, leading to a total of 65536 simulated paths per method.

The Mersenne Twister results yields a mean estimated option price of 0.19575, compared to the theoretical price of 0.19715. The average absolute error across trials is 0.00597, with a maximum absolute error of 0.01702. The variance of the estimated prices is 0.00005398, corresponding to a standard deviation of 0.00735. These results indicate a relatively tight distribution of price estimates around the theoretical value.

In contrast, simulations using the Sobol' sequence produced a mean estimated option price of 0.23363, which deviates from the theoretical price of 0.19715 by an absolute difference of 0.03648. Despite using a large number of simulated paths (65536), this level of discrepancy is considered suboptimal, as such sample sizes are typically expected to yield tighter convergence. The average absolute error across trials is 0.04617, and the maximum absolute error reaches to 0.33548. The price estimates shows a variance of 0.00755007 and a standard deviation of 0.08689. Furthermore, the observed minimum and maximum prices (0.18357 and 0.53263, respectively) indicate a much wider dispersion compared to the Mersenne Twister results, reflecting greater instability and less reliable convergence behavior.

This discrepancy in performance prompted a deeper investigation into the underlying causes of the observed deficiencies in the Sobol' sequence simulations. To this end, the next section conducts a series of statistical analyses aimed at diagnosing potential issues inherent to the structure of Sobol-generated paths. In particular, we focus on two key diagnostic tools: moment analysis, to evaluate the marginal distributions of individual Sobol' dimensions, and correlation analysis, to detect any unexpected dependencies between dimensions that could impair the sequence's uniformity and effectiveness.

ROOT CAUSE ANALYSIS OF THE OBSERVED DISCREPANCIES

Moment Analysis of Sobol' Sequences

In order to better understand the statistical properties of Sobol' sequences in high-dimensional simulations, we perform a detailed investigation of the first four moments --- mean, variance, skewness, and kurtosis --- of the sequences. Two cases are analyzed: the raw Sobol' sequences on $[0,1)$, and their transformation into standard normal space via the inverse cumulative distribution function (ICDF), Φ^{-1}.

We generated samples at sizes 2^{12}, 2^{16}, 2^{20}, 2^{24} and 2^{28}, corresponding to 4096, 65536, 1048576, 16777216, and 268435456 samples, respectively.

For the raw Sobol' samples (i.e., before applying ICDF), the results are summarized below and depicted in Table 3.

- The sample mean converges to 0.5 as the number of samples increases, aligning with the expected value for a uniform distribution.
- The sample variance also converges to $\frac{1}{12} \approx 0.08333333$, the theoretical variance of the uniform distribution.
- The skewness remaines effectively zero across all sample sizes, indicating symmetric distributions.
- The kurtosis stabilizeds at 1.8, as theoretically expected for uniform random variables.

Table 3. Moments of Raw Sobol' Sequence $([0,1))$

n_{samples}	Mean	Variance	Skewness	Kurtosis
2^{12} (4096)	0.499969	0.08333329	1.06×10^{-10}	1.79999865
2^{16} (65536)	0.499992	0.08333333	6.29×10^{-12}	1.79999999
2^{20} (1048576)	0.499999	0.08333333	2.74×10^{-14}	1.8
2^{24} (16777216)	0.499999	0.08333333	1.08×10^{-16}	1.8
2^{28} (268435456)	0.5	0.08333333	-2.77×10^{-19}	1.8

After applying the inverse cumulative normal function Φ^{-1} to transform the Sobol' points to standard normal deviates, the moments shown in Table 4 are observed to behave as follows:

- The mean converges rapidly towards 0, as expected for a standard normal distribution.

- The variance converges to 1, as theoretically expected.
- The skewness remaines near 0, indicating symmetry of the resulting normal distributions.
- The kurtosis approaches 3, matching the kurtosis of a standard normal distribution.

Table 4. Moments of Transformed Sobol' Sequence $(\mathcal{N}(0,1))$

n_{samples}	Mean	Variance	Skewness	Kurtosis
2^{12} (4096)	-2.23×10^{-4}	0.99908158	−0.002273	2.97942869
2^{16} (65536)	-5.99×10^{-5}	0.99993473	−0.000829	2.99790646
2^{20} (1048576)	-4.45×10^{-6}	0.99999588	-8.43×10^{-5}	2.9998241
2^{24} (16777216)	-3.11×10^{-7}	0.99999974	-7.54×10^{-6}	2.99998616
2^{28} (268435456)	-2.13×10^{-8}	0.99999998	-6.30×10^{-7}	2.99999896

These empirical findings align with the theoretical properties established in the literature.

Correlation Analysis in Sobol' Sequences

It is well-known that Sobol' sequences can exhibit substantial correlations between different dimensions, meaning that the generated quasi-random numbers in different dimensions are correlated. In the context of this study, references to inter-dimensional correlation specifically refer to the correlation between the quasi-random numbers assigned to different dimensions. This phenomenon is already mentioned in the early works on low-discrepancy sequences (Sobol', 1967). In (Sobol' et al., 2012), it is noted that, for a particular implementation of Sobol' numbers, "a test done with 2500 dimensions showed that 2449 pairs of consecutive dimensions have correlation greater than 70% (in absolute value)."

In our study, with 21201 dimensions and 4096 samples, we perform a similar analysis. Among the $\frac{21201 \times 21200}{2} = 224,730,600$ possible distinct dimension pairs, we observe that 415,391 pairs exhibit a correlation greater than 0.1, corresponding to approximately 0.18% of all pairs. Furthermore, 91,668 pairs show correlations exceeding 0.5, and 91,631 pairs exceed 0.6, both representing approximately 0.04% of all possible pairs. When considering even stronger correlations, 91,493 pairs show correlations greater than 0.7, but this number dropped sharply to only 110 pairs when the threshold is raised to 0.8, accounting for a negligible fraction of

0.00005% and highlighting that while moderate correlations are relatively common, extremely strong correlations remain very rare. Specifically among consecutive dimensions, we identified 10 pairs with correlation above 0.7. This is in stark contrast to the findings of (Sobol' et al., 2011), where nearly all consecutive pairs in their test showed high correlation—highlighting the improved behavior of the *Scipy Sobol* implementation in high-dimensional settings.

This strong correlation behavior in Sobol' sequences contrasts sharply with the performance of pseudo-random generators such as Mersenne Twister. In a comparable experiment using Mersenne Twister with 4096 samples across 21201 dimensions, the maximum absolute correlation observed is only 0.091 (between dimensions 15689 and 15966).

Additionally, it is important to highlight that the magnitude of these correlations in Sobol' sequences diminishes significantly as the number of samples increases. For instance, considering the dimension pair (1229, 6014), the correlation is $\rho = 0.9375$ with 4096 samples but drops dramatically to $\rho = 0.0249$ when the number of samples are increased to 20000. This behavior is consistent with the theoretical expectations regarding the asymptotic behavior of Sobol' sequences.

Some representative scatter plots of correlated dimension pairs are shown in Figure 2. These visualizations illustrate how severe clustering can occur when the random variables associated with different dimensions are strongly correlated.

Following the application of the inverse standard normal transformation to the Sobol' samples, we performed a similar correlation analysis. Among the 224,730,600 possible distinct dimension pairs, 847,904 pairs exhibit correlations greater than 0.1, corresponding to approximately 0.38% of all pairs. Furthermore, 91,668 pairs show correlations exceeding 0.5, while 91,394 pairs exceed 0.6, both representing approximately 0.04% of all possible pairs and indicating that moderate levels of dependency remain relatively prevalent. However, as the correlation threshold is increased beyond 0.6, a sharp decline is observed: only 189 pairs exhibited correlations greater than 0.7, accounting for a minuscule proportion of approximately 0.00008% of all pairs. This sharp drop between 0.6 and 0.7 thresholds differs slightly from the pattern observed before the transformation, where a comparable decline is only observed between 0.7 and 0.8 thresholds. These findings suggest that the inverse transformation modifies the underlying dependency structure of Sobol' sequences, effectively reducing the prevalence of very high correlations (greater than 0.7) and shifting dependencies into the correlation range (above 0.6).

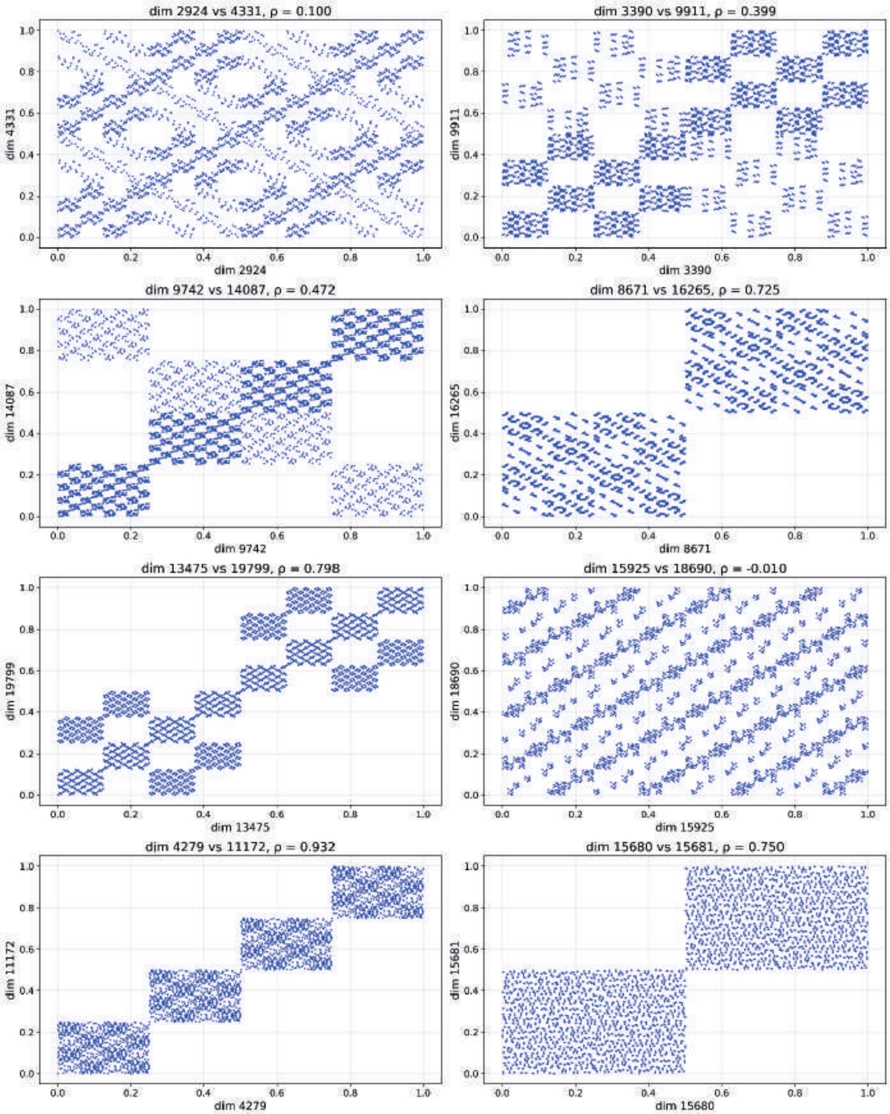

Figure 2. Scatter plots of selected dimension pairs from Sobol' sequence.

Variance Bias in Incremental Construction

In classical Monte Carlo simulation of Brownian motion, the standard discretization method constructs the terminal value W_T by sequentially summing small increments generated from independent Gaussian variables.

This method, commonly referred to as the incremental construction, approximates W_T as:

$$W_T^I = \sum_{i=1}^{N} \sqrt{dt}\, z_i \#(14)$$

where $z_i \sim \mathcal{N}(0,1)$ are independent standard Gaussian random variables, N is the number of time steps, and $dt = T / N$.

In an ideal Monte Carlo settings, if the z_i are truly independent, the following properties would hold:

$$E[z_i] = 0, \quad \forall i \#(15)$$

$$\mathrm{V}[z_i] = 1, \quad \forall i \#(16)$$

$$E[z_i z_j] = 0, \quad \text{for } i \neq j \#(17)$$

where $(E[\cdot])$ and $(V[\cdot])$ denote the expectation and variance operators, respectively. Consequently, the mean and variance of (W_T^I) satisfy:

$$\mathrm{E}[W_T^I] = 0 \#(18)$$

$$V[W_T^I] = T \#(19)$$

However, when quasi-random sequences like Sobol' numbers are used to generate z_i via the inverse normal CDF, strict independence between coordinates is no longer guaranteed. Particularly at small sample sizes or high dimensions, residual correlations between z_i and $z_j\,(i \neq j)c$ an introduce a bias into the variance of W_T^I.

To quantify this effect, assume that the empirical variance of each z_i is $v \lesssim 1$, and that the empirical correlation between different dimensions is $\tilde{n}_{ij} = E[z_i z_j] - E[z_i]E[z_j]$. Then, the variance of W_T^I becomes:

$$V[W_T^I] = \sum_{i=1}^{N} dt\, V[z_i] + 2\sum_{i<j} dt\, \rho_{ij} \#(20)$$

$$= T\left(v + \frac{2}{N}\sum_{i<j} \rho_{ij}\right) \#(21)$$

We define the variance distortion term C induced by the correlations as:

$$C = \frac{2}{N}\sum_{i<j} \rho_{ij} \#(22)$$

such that the total variance is expressed as $T(v + C)$.

To complement the theoretical analysis, we computed the variance v, the term C caused by inter-dimensional correlations, and the total variance $v + C$ for both Sobol' and Mersenne Twister sequences across different dimensions. The sample size is fixed at 4096 simulations. The tested dimensionalities includes n_{dims} = 100, 500, 1000, 2000, and 21201, covering a wide range from low to extremely high dimensions. In Table 5, the empirical results are summarized. As observed, the Sobol' sequences exhibit significant deviations in the $v + C$ values, particularly for high dimensions such as $n_{\text{dims}} = 21201$, where $v + C$ exceeds 4.0. This reflects the impact of dimension-dependent correlations inherent to Sobol' sequences. In contrast, Mersenne Twister sequences show relatively stable behavior, with $v + C$ values remaining close to 1 even at high dimensions, confirming their low-correlation pseudo-random nature.

Table 5. Variance and variance distortion for Sobol' and Mersenne Twister sequences at $n_{\text{samples}} = 4096$

Method	n_{dims}	v	C	$v + C$
Sobol	100	0.999275	-0.044254	0.955021
Sobol	500	0.999275	-0.141961	0.857314
Sobol	1000	0.999278	-0.308172	0.691105
Sobol	2000	0.999277	-0.386930	0.612347
Sobol	21201	0.999282	3.061985	4.061268
MT	100	0.998997	-0.013125	0.985872
MT	500	0.998894	0.014969	1.013864
MT	1000	0.998440	-0.003114	0.995326
MT	2000	0.998587	-0.018400	0.980186
MT	21201	0.999428	0.001260	1.000688

Effect of Scrambling, Skipping, and Brownian Bridge on Variance Stability

In the previous sections, we observed that in the absence of any enhancements, Sobol' sequences exhibited significant deterioration in the total variance $v + C$ as the dimensionality increased. In this section, we explore how three different techniques — scrambling, skipping initial points, and Brownian Bridge construction — affect the variance stability.

Variance Stability under Scrambling

When scrambling is enabled, Sobol' sequences show a remarkable improvement in variance properties. As shown in Table 6, even for $n_{\text{dims}} = 21201$, the total variance $v + C$ is corrected to approximately 1.0025. Across all tested dimensions (n_{dims} = 100, 500, 1000, 2000, 21201), the deviations of $v + C$ from 1 remain within ± 0.006 margins, indicating stability.

Thus, the results observed in our tests validate the theoretical expectations: scrambling substantially improves the variance stability of Sobol' sequences, making them highly reliable even when applied to very high-dimensional integration problems.

Variance Stability under Initial Points Skipping

Applying a large skip of $2^{19} = 524,288$ points (without scrambling) results in a notable improvement in the stability of total variance estimates. For lower dimensions (n_{dims} = 100, 500), the total variance $v + C$ remains very close to the theoretical value of 1, with deviations on the order of 10^{-3} or less. Even in extremely high-dimensional settings, such as n_{dims} = 21201, the total variance remains stable around 0.9934 (see Table 6), representing a substantial improvement over the non-skipped case.

These results suggest that skipping a substantial number of initial Sobol' points can also substantially improve numerical behavior across a wide range of dimensions. In our tests, skipping is consistently observed to have a positive impact on stability, supporting the understanding that Sobol' sequences tend to perform better when an initial portion of the sequence is discarded.

Several studies, including Owen (Owen, 2021), have pointed out that skipping initial points in *scrambled* Sobol' sequences may disrupt the underlying randomized net structure, potentially degrading convergence. While our findings demonstrate the practical benefits of skipping in *non-scrambled* settings, further research is needed to better understand how skipping interacts with scrambling in different simulation contexts, particularly with respect to determining how many points should be skipped, which may vary significantly depending on the integrand, dimensionality, and target accuracy.

Variance Stability under Brownian Bridge Construction

Brownian bridge (BB) construction is a widely used technique to improve the efficiency of path generation in Monte Carlo and Quasi-Monte Carlo simulations. By carefully redistributing the variance contributions across dimensions, BB can significantly stabilize the numerical behavior, particularly in high-dimensional settings.

Consider a standard Brownian motion process $W(t)$ over the interval $[0,T]$, with fixed endpoints:

$W(0)=0$

$W_T^{\mathrm{BB}}=\sqrt{T}z_1$, where $z_1 \sim \mathcal{N}(0,1)$

It follows that $E\left(W_T^{\mathrm{BB}}\right)=0$ and $V\left(W_T^{\mathrm{BB}}\right)=T$ since $E(z_1)=0$ and $V(z_1)=1$.

Intermediate values of the Brownian motion are not generated sequentially in time. Instead, the process recursively fills in midpoints of the largest remaining intervals. Suppose we already have values at two time points T_i and T_{i+1}, and we want to generate the value at a time $t \in (T_i, T_{i+1})$. This is done using:

$$W(t)=\frac{T_{i+1}-t}{T_{i+1}-T_i}W(T_i)+\frac{t-T_i}{T_{i+1}-T_i}W(T_{i+1})+\sqrt{\frac{(T_{i+1}-t)(t-T_i)}{T_{i+1}-T_i}}\cdot z_k \#(23)$$

where each $z_k \sim \mathcal{N}(0,1)$ is an independent standard normal random variable.

The first two terms represent a linear interpolation, while the third term introduces stochasticity consistent with Brownian motion's properties. In theory, when ideal random numbers are used, the Brownian Bridge construction should maintain stability across dimensions without degradation as dimension increases. Minor deviations from the ideal variance are primarily attributed to residual correlations among quasi-random samples. That is, while Brownian bridge organizes the variance efficiently, quasi-random sequences like Sobol' are not perfectly independent across dimensions, and small correlations can slightly affect variance. See (Sobol' and Kucherenko, 2005, Kucherenko and Shah, 2007 and Bianchetti et al., 2015) for more information.

In our numerical experiments, we observe that when Brownian Bridge construction is applied to Sobol' sequences, the total variance $v+C$ remains very close to 1 across all dimensions (see Table 6). Even at $n_{\mathrm{dims}}=21201$, the total variance is approximately 0.9989, reflecting agreement with theoretical expectations. These results validate that Brownian Bridge dramatically improves variance stability in high-dimensional settings, and small observed deviations are consistent with the minor correlation effects inherent to quasi-random sequences rather than flaws in the Brownian Bridge algorithm itself.

Table 6. Summary of Total Variance $v+C$ Under Different Enhancements

Method	**Scrambling Enabled**			
	Dimensions	**Variance**	**Distortion**	**Total**
Sobol	100	1.000169	0.002468	1.002636
Sobol	500	0.999994	-0.015104	0.984890
Sobol	1000	0.999974	0.006229	1.006202
Sobol	2000	1.000021	0.003872	1.003893
Sobol	21201	0.999992	0.002492	1.002484

Table 6. continued

Method	**Skip = 2^{19} (524288 points)**			
	Dimensions	**Variance**	**Dimensions**	**Total**
Sobol	100	1.000097	0.001626	1.001723
Sobol	500	1.000026	-0.000100	0.999926
Sobol	1000	1.000002	-0.004902	0.995100
Sobol	2000	1.000004	-0.014498	0.985507
Sobol	21201	0.999997	-0.006642	0.993355
Method	**Brownian Bridge Enabled**			
	Dimensions	**Variance**	**Distortion**	**Total**
Sobol	100	0.999275	-0.000388	0.998886
Sobol	500	0.999275	-0.000234	0.999040
Sobol	1000	0.999278	0.000012	0.999290
Sobol	2000	0.999277	-0.000324	0.998953
Sobol	21201	0.999282	-0.000391	0.998892

EFFECT OF SCRAMBLING, SKIPPING, AND BROWNIAN BRIDGE ON ASIAN OPTION PRICING

To evaluate the practical impact of various Sobol' sequence enhancement techniques on financial simulations, we conducted a series of controlled experiments focusing on the pricing of a Geometric Asian option. The theoretical price, previously established in Section "geometric asian option pricing", serves as a benchmark for assessing pricing accuracy.

In this study, we specifically tested the effects of scrambling, initial skipping, and Brownian bridge construction. Four different Sobol configurations are examined:

- *Baseline*: No scrambling, no Brownian Bridge.
- *Scrambled Sobol*: Scrambling enabled, no Brownian Bridge.
- *Brownian Bridge*: No scrambling, Brownian Bridge enabled.
- *Large Skip Only*: Skip = 2^{19} (524,288 points), No scrambling, no Brownian Bridge.

Each configuration, except the initial skip case, is evaluated over 16 independent trials with 4096 paths per trial to ensure statistical consistency. The initial skip case configuration used a single batch of 4096 paths without repetition across trials.

The results, summarized in Table 7, reveal clear and consistent trends regarding the effectiveness of these enhancement techniques. Additionally, results for the standard Mersenne Twister simulation have been included in the table to provide reference for comparison with theoretical MC.

The baseline configuration, without scrambling or Brownian bridge, performs poorly: the mean estimated price is 0.23363, deviating significantly from the theoretical value. The maximum absolute error reaches to 0.33548, and the variance across trials is as high as 0.00755. These large errors are primarily attributed to structural artifacts and correlations inherent in the raw Sobol' sequence.

Enabling scrambling leads to a substantial improvement. The mean price converges to 0.19775, with a maximum error of only 0.00908, and The variance is reduced significantly, reaching approximately to 1.542×10^{-5} . This supports theoretical findings that scrambling improves uniformity, reduces bias, and allows for effective variance estimation.

Applying Brownian bridge construction without scrambling further enhances performance. The mean price achieved is 0.19711, with a maximum absolute error of just 0.00121, and the variance is reduced to 2.4×10^{-7} . This dramatic variance stabilization is consistent with the theoretical expectation that Brownian bridge reallocates variance contributions, improving the convergence behavior, particularly in high-dimensional settings.

Finally, applying a large skip of 2^{19} points—without scrambling or Brownian bridge—also produces notably accurate results. The mean price is 0.19608, and the maximum absolute error is just 0.00107, indicating a meaningful reduction in simulation bias. While skipping alone may not match the variance stabilization achieved by Brownian bridge, it remains an effective and simple strategy for improving coverage and reducing structural artifacts in Sobol'-based simulations.

Table 7. Comparison of Different Configurations in Geometric Asian Option Pricing

Configuration	Mean Price	Avg Error	Max Error	Variance
MT	0.19575	0.00597	0.01702	0.00005398
No Scramble, No BB	0.23363	0.04617	0.33548	0.00755007
Scramble = True, No BB	0.19775	0.00319	0.00908	0.00001542
BB = True, No Scramble	0.19711	0.00041	0.00121	0.00000024
Skip = 2^{19} (1 batch)	0.19608	0.00107	0.00107	N/A

In summary, our experiments demonstrate that proper configuration of Sobol' sequences is crucial for achieving high precision in QMC simulations in finance. Scrambling significantly reduces bias and variance, while Brownian Bridge construction further stabilizes variance by optimizing the allocation of variability across dimensions. Initial skipping alone can offer measurable improvements, but the best results are achieved when Brownian Bridge technique are employed. These findings reinforce the necessity of combining enhancement strategies to fully exploit the potential of quasi-Monte Carlo methods in high-dimensional option pricing problems.

GPU-ACCELERATED SIMULATIONS

The use of GPUs in computational finance has been extensively explored in the literatüre (Dempster et al., 2018). The inherently parallel structure of Monte Carlo simulations for path-dependent option pricing makes them well-suited for GPU-enabled parallel computing frameworks like CUDA. In our study, we harness the parallel processing power of GPUs to enhance the speed and efficiency of option pricing computations.

In our implementation, we primarily utilize the CuPy library to perform all array operations, random number transformations, and Monte Carlo path simulations on the GPU. CuPy provides a highly efficient, NumPy-compatible interface that allows straightforward migration of CPU-based codes to CUDA-enabled devices with minimal adjustments. PyTorch is employed exclusively for generating Sobol' sequences directly on the GPU, as CuPy currently lacks a native GPU-based Sobol generator. By using PyTorch's *Sobol Engine* for quasi-random number generation and relying on CuPy for the remaining computational tasks, we combine the strengths of both libraries to maximize performance and maintain numerical accuracy in high-dimensional Monte Carlo simulations.

This section presents a comparative evaluation of GPU-accelerated Monte Carlo and Quasi-Monte Carlo simulations in the context of pricing Geometric

Asian options (BS framework). To assess the impact of GPU acceleration, we conducted a series of controlled experiments focusing on runtime performance. Specifically, we benchmarked CPU-based versus GPU-based implementations across three scenarios: Mersenne Twister random number generation, Sobol' sequence generation, and Sobol' sequence generation combined with Brownian Bridge construction. These experiments are designed to isolate and quantify the computational advantages offered by GPU parallelization while keeping the pricing methodology and simulation parameters consistent. In all cases, CPU implementations are executed serially and serve as a baseline for evaluating the speedup and efficiency improvements achieved through GPU parallelization.

Algorithm 2 (see Figure 3) models the core simulation loop for Monte Carlo and Quasi-Monte Carlo simulations, where random numbers are either sampled from a standard normal distribution (for Mersenne Twister) or generated via Sobol' sequences followed by an inverse transformation. Each path is constructed through cumulative summation of the simulated increments and subsequently used to compute the option payoff. Algorithm 3 (see Figure 4) specifically describes the construction of Brownian Bridge increments using Sobol' sequences, to allocate early Sobol' dimensions to the most critical parts of the simulated Brownian motion path.

Algorithm 2 Per-thread vectorized path simulation using either Mersenne Twister or Sobol' sequences

$dt \leftarrow T/N$
$drift \leftarrow (r - 0.5\sigma^2) \cdot dt$
$vol \leftarrow \sigma \cdot \sqrt{dt}$
if Generator = Mersenne Twister **then**
  $Z \leftarrow$ StandardNormal[PATHS][N] ▷ Generated using `cupy.random`
else if Generator = Sobol **then**
  $U \leftarrow$ Sobol[PATHS][N] ▷ From `torch.quasirandom.SobolEngine`
  $Z \leftarrow \sqrt{2} \cdot \texttt{erfinv}(2U - 1)$ ▷ Inverse CDF transform
end if
$increments \leftarrow drift + vol \cdot Z$ ▷ Element-wise vectorized operation
$log\_paths \leftarrow \texttt{cumsum}(increments, \text{axis} = 1)$ ▷ Sequential in time, parallel across paths
$log\_paths \leftarrow \texttt{concat}([0], log\_paths)$ ▷ Insert $W_0 = 0$
$S \leftarrow S_0 \cdot \exp(log\_paths)$ ▷ Path prices
$GA \leftarrow \exp\left(\frac{1}{N}\sum_{i=1}^{N} \log S_i\right)$ ▷ Geometric mean per path
$payoff \leftarrow \max(GA - K, 0)$ ▷ Vectorized payoff calculation
$price \leftarrow \exp(-rT) \cdot \texttt{mean}(payoff)$

Figure 3. Illustrative figure of per-thread vectorized path simulation workflow using Mersenne Twister or Sobol' sequences

Algorithm 3 Brownian Bridge construction using Sobol-generated normal deviates

$U \leftarrow$ Sobol[PATHS][N] ▷ Sobol samples from `torch.quasirandom.SobolEngine`
$Z \leftarrow \sqrt{2} \cdot \texttt{erfinv}(2U - 1)$ ▷ Inverse CDF transform
Initialize $W_t[n] \leftarrow \sqrt{T} \cdot Z[t][0]$ ▷ Set terminal value
Build midpoint schedule using recursive bisection
$cur\_z \leftarrow 1$
for each midpoint (i, l, r) **do**
    $\Delta \leftarrow t_r - t_l$
    $a \leftarrow \frac{(t_r - t_i) \cdot W_t[l] + (t_i - t_l) \cdot W_t[r]}{\Delta}$
    $b \leftarrow \sqrt{\frac{(t_i - t_l)(t_r - t_i)}{\Delta}}$
    $W_t[i] \leftarrow a + b \cdot Z[t][cur\_z]$
    $cur\_z \leftarrow cur\_z + 1$
end for
$dw[t][i] \leftarrow (W_t[i+1] - W_t[i]) \cdot \sqrt{N/T}$ ▷ Construct Brownian increments

Figure 4. Illustration of Brownian Bridge path generation based on Sobol-normal samples

We also remark that Table 8 summarizes the execution times recorded for each simulation setup under a single batch execution.

Table 8. Execution time comparison for MC and QMC simulations (in seconds)

Method	Implementation	Time (s)
MT	CPU	3.148
MT	GPU	*0.095*
Sobol	CPU	4.889
Sobol	GPU	*0.570*
Sobol + BB	CPU	71.266
Sobol + BB	GPU	*5.788*
Sobol + Scramble	CPU	5.582
Sobol + Scramble	GPU	*1.085*
Sobol + (Skip = 2^{19})	CPU	18.108
Sobol + (Skip = 2^{19})	GPU	*13.552*

The results in Table 8 highlight the considerable computational advantages offered by GPU acceleration across various simulation methods. Among these, the most significant improvement is observed in standard Monte Carlo simulations using Mersenne Twister, where the GPU implementation completes the task in just 0.095 seconds, compared to 3.148 seconds on the CPU—a 33-fold speedup. This dramatic reduction demonstrates the efficiency of GPU-based parallel random number generation for large-scale simulations.

We next examine the performance of quasi-Monte Carlo simulations using Sobol' sequences. The GPU implementation achieves a runtime of 0.57 seconds, offering an 8.5-fold speedup relative to its CPU counterpart. This improvement underscores the effectiveness of GPU acceleration even when using low-discrepancy sequences, which are traditionally more structured and less amenable to parallelism than pseudo-random number generators. In the scrambled Sobol case, GPU runtime increases modestly to 1.085 seconds due to the additional computational cost of digital scrambling, yet it remains significantly faster than CPU execution.

Brownian Bridge construction, when applied alongside Sobol' sequences, introduces additional computational overhead due to its recursive midpoint structure. Nonetheless, the GPU implementation reduces runtime from 71.266 seconds on the CPU to 5.788 seconds, achieving a 12-fold acceleration. Despite this gain, BB simulations remain more time-consuming overall, as the recursive dependencies inherently limit parallelism on GPU architectures.

A more detailed breakdown of execution times reveals the primary computational bottlenecks in each configuration. For Sobol' simulations on GPU, approximately 86% of the total time is spent on generating quasi-random numbers, while the remaining time is used for path construction. In the scrambled Sobol case, the extra 0.515 seconds of overhead stems from scrambling operations. In contrast, applying a large skip (e.g., 2^{19}) leads to a total runtime of 13.552 seconds on GPU, indicating that while skipping improves sequence quality, it is computationally inefficient in practice.

In Brownian Bridge simulations, the performance bottleneck shifts away from number generation. Approximately 90% of the GPU runtime is spent on recursive midpoint interpolation, while only about 8% is used for generating the Sobol' sequence. This shift clearly shows that the recursive structure of BB—not the sampling method—is the dominant contributor to total execution time in this configuration.

These observations suggest that the combination of CuPy and PyTorch---both high-level GPU libraries---offers an effective and practical solution for accelerating Monte Carlo and Quasi-Monte Carlo simulations without requiring low-level custom CUDA kernel programming. High-level libraries like CuPy and PyTorch handle kernel generation and GPU memory management automatically, whereas low-level CUDA programming requires manually writing and optimizing custom kernels. Using such high-level libraries allows for quick implementation while maintaining sufficient computational performance for pricing single exotic options, such as the Geometric Asian option considered in this study.

However, it is important to note that this analysis focuses on a single product type. In scenarios where portfolios of multiple exotic options are to be priced, or risk metrics such as Value-at-Risk (VaR) and Conditional Value-at-Risk (CVaR) are to be computed, the overall dimensionality (i.e., the total number of time steps × assets) —and thus the computational complexity— can become extremely large. In such cases, relying solely on high-level libraries may no longer be sufficient, and more optimized implementations involving explicit kernel configurations could be necessary to fully exploit the available GPU resources.

Lastly, the remarkable speedup observed with GPU-accelerated Mersenne Twister simulations suggests that further studies combining Mersenne Twister random number generation with advanced variance reduction techniques under full GPU parallelization could make valuable contributions to the computational finance literature. Moreover, exploring algorithmic modifications to the Brownian Bridge construction that improve its compatibility with parallel architectures presents another promising direction for enhancing the efficiency of quasi-Monte Carlo methods.

CONCLUSION

This study provides a comprehensive computational analysis of Quasi-Monte Carlo methods using Sobol' sequences in comparison to traditional Monte Carlo simulations based on the Mersenne Twister generator. While Sobol' sequences are theoretically known to offer superior convergence rates, our investigation reveals notable challenges when applying them to high-dimensional problems—arising, for instance, in financial simulations such as Geometric Asian option pricing.

The theoretical background and related literature highlight the known strengths of Sobol' sequences, but also hint at their sensitivity to dimension ordering and structural artifacts. Through one- and multi-dimensional integral tests, we validate the accuracy and convergence behavior in integral calculations using Sobol' sequences. However, when transitioning to the high-dimensional setting of Asian option pricing, significant deviations are observed, especially in baseline Sobol configurations.

To investigate the source of the observed discrepancies, we conduct detailed moment and correlation analyses; they reveal a persistent bias inherent in the incremental construction of Sobol' sequences. This bias is found to be effectively mitigated through various enhancement techniques, including scrambling, initial skipping, and Brownian Bridge construction. Among these, Brownian bridge yields the most accurate option price estimates, while scrambling offers both improved accuracy and straightforward parallel implementation. Although initial skipping alone provides noticeable improvements, its overall effectiveness is more limited relative to Brownian Bridge technique.

Finally, we implement GPU-accelerated versions of all simulation methods, achieving substantial speedups across all configurations. The most striking gains are observed in Mersenne Twister-based simulations, where GPU parallelism yields a 33-fold runtime reduction of the base case. Sobol with Brownian Bridge simulations also benefits significantly from GPU acceleration, though to a lesser extent due to algorithmic limitations in parallelizing recursive path construction.

Overall, our findings reinforce the importance of proper configuration when applying QMC methods to high-dimensional problems. Moreover, they highlight the combined value of theoretical insight, algorithmic enhancement, and hardware-level acceleration in achieving both numerical precision and computational efficiency in modern financial simulations.

FUTURE WORK

The findings of this study suggest several directions for future research that could further enhance the efficiency, accuracy, and applicability of GPU-accelerated Monte Carlo and Quasi-Monte Carlo methods in computational finance.

First, the remarkable performance gains achieved through GPU-accelerated Mersenne Twister (MT) simulations indicate a valuable opportunity for further enhancement. In particular, the use of high-level GPU libraries such as CuPy enables the realization of substantial speedups without requiring low-level CUDA programming, making efficient pseudo-random number generation easily accessible within Python environments. While MT-based simulations offer excellent computational speed due to their lightweight nature, they are not inherently variance-reducing. Therefore, integrating MT with advanced variance reduction techniques—such as control variates, stratification, or antithetic sampling—under full GPU parallelization could substantially improve simulation accuracy without sacrificing computational efficiency. This combination may serve as a practical and scalable alternative to Quasi-Monte Carlo methods in applications.

Second, although the Brownian Bridge construction is a well-established variance reduction technique, its recursive nature limits full parallelization, especially on GPU architectures. One fruitful direction for future work would be to investigate alternative algorithmic structures or approximations that retain the variance allocation benefits of Brownian Bridge while reducing memory bottlenecks or enabling greater parallel throughput. Additionally, it would be worth exploring whether custom low-level CUDA kernel implementations—specifically designed to optimize memory access and thread scheduling—could further enhance the performance of Brownian Bridge-based simulations beyond what is achievable with high-level GPU libraries alone.

Third, some studies, particularly in the context of *scrambled* Sobol' sequences, caution that skipping initial points may interfere with the randomized net structure and degrade convergence (Owen, 2021). Nonetheless, other works such as (Radović et al., 1996) report reduced integration error when early low-quality points are avoided. Our results support this view in the *non-scrambled* case: skipping a large number of initial points (e.g., 2^{19}) improved accuracy in high-dimensional simulations without Brownian Bridge. In addition to the interaction between skipping and scrambling, the question of how many points should be skipped remains problem-dependent, underscoring the need for more systematic analysis across different integrands and dimensionalities to better understand when skipping improves or degrades performance.

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Appendix

GPU and CUDA specifications

CUDA toolkit version 12.6 was used for all of the simulations. The online documentation for this version is available at (https://docs.nvidia.com/cuda/archive/12.6.0/index.html). All simulations were conducted on a single NVIDIA GeForce RTX 4070 Laptop GPU. The specifications of the device are summarized in Table 9.

Table 9. Hardware specifications of the GPU used in simulations (NVIDIA GeForce RTX 4070)

Title	Title
Device Name	NVIDIA GeForce RTX 4070 Laptop GPU
CUDA Driver / Runtime Version	12.6 / 12.6
Compute Capability	8.9
CUDA Cores	4608
Global Memory	8 GB (8585 MB)
Shared Memory per Block	49 KB
Constant Memory	64 KB
L2 Cache Size	32 MB
Memory Clock Rate	8001 MHz
Memory Bus Width	128-bit
GPU Max Clock Rate	2175 MHz
Maximum Texture Dimension (1D)	131072
Maximum Texture Dimension (2D)	(131072, 65536)
Maximum Texture Dimension (3D)	(16384, 16384, 16384)
Maximum Threads per Multiprocessor	1536
Maximum Threads per Block	1024
Max Block Dimensions (x, y, z)	(1024, 1024, 64)
Max Grid Dimensions (x, y, z)	(2147483647, 65535, 65535)
Warp Size	32
Support for Concurrent Copy and Execution	Yes
Unified Memory (UVA) Support	Yes
ECC Support	No

Python Environment and Library Versions

All simulations were implemented using Python 3.12.4, supported by a set of high-performance numerical and GPU-accelerated libraries. The core packages and their versions are summarized in Table 10.

Table 10. Python Environment and Library Versions

Library	Version
Python	3.12.4
NumPy	1.26.4
SciPy	1.13.1
CuPy	13.3.0
PyTorch	2.5.1

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to this work.

Chapter 3

An Turkey's Biodiesel Potential and the Economics of Related Agricultural Products

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Abstract

With the rapid increase in the world population and the intensification of industrialization, the need for energy to sustain human life is also increasing day by day. As the use of energy resources has reached a significant level today, environmental pollution, greenhouse gas emissions, and the resulting climate change, along with the limited capacity of raw materials and energy sources, have led people to seek alternative energy sources.

Renewable energy sources are becoming increasingly important due to their low cost, minimal environmental impact, and very low levels of greenhouse gas emissions. These renewable energy sources include solar, wind, geothermal, and biomass energy.

Biomass is an energy source derived from all natural materials of non-fossil origin, including plant- and animal-based matter. The most important characteristics of biomass are its environmental friendliness and sustainability.

Biodiesel is a biomass-derived energy source in liquid form. It is obtained through processing oils derived from oilseed crops as well as used cooking oils.

This study examines the production quantities and cultivation areas of vegetable oils used in biodiesel production, along with the import and export statistics of these products in Turkey. Based on these data, the potential of biodiesel production and its evaluation have been assessed.

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1. Introduction

As a result of rapid population growth and industrialization worldwide, the demand for energy—an essential input for sustaining human life—is steadily increasing. Since the 1970s, energy consumption across all sectors has become one of the most critical issues globally. Following the oil crises of 1973 and 1979, countries around the world began to show increased interest in energy conservation measures. In the 1980s, attention shifted primarily to environmental pollution caused by the use of fossil fuels.

In recent years, energy consumption, greenhouse gas emissions, and their potential impacts on global climate change have become some of the most widely discussed topics. One of the most effective ways to reduce energy consumption in industry, transportation, commerce, housing, and agriculture is by improving energy efficiency.

Today, the use of energy and other natural resources has reached a significant level in the industrial world. As a result, the availability of natural resources has started to decline on one hand, while environmental degradation, such as pollution, continues to increase on the other. Moreover, technical improvements in energy conversion have not been implemented effectively enough.

In both developed and developing countries, several factors must be considered to determine future levels of energy production and consumption. These include population growth, economic productivity, consumer behavior, and technological advancements. Öztürk, H. H. (2006)

Despite the limited capacities of raw materials and energy resources, the continuous and rapidly increasing demand for both, along with the finite reserves of primary energy sources, compels humanity to seek alternative and non-conventional energy sources. In addition to the limited reserves of primary energy sources, factors such as rising fuel prices, population growth, industrialization, the necessity of utilizing national resources, the negative environmental impacts of conventional fuels, and the issue of climate change make the use of renewable energy sources essential within the scope of new energy technologies. Kapluhan, E. (2014)

Atmospheric carbon dioxide levels have been steadily increasing since the beginning of the Industrial Revolution, and this trend is expected to accelerate further as the global economy continues to grow. Significant climate changes are strongly associated with the rising atmospheric concentrations of certain gases, particularly carbon dioxide. Compared to fossil fuels, renewable energy technologies generate very low or near-zero greenhouse gas emissions. Çoban, O. R. H. A. N., & Kılınç, N. Ş. (2016)

The importance of renewable energy sources is increasingly growing today as an alternative to fossil-based fuels. The renewable nature of these sources, their lower costs, and the relatively minimal negative environmental impacts associated with their use contribute to the rising demand. Solar energy in particular, along with wind, geothermal, and biomass energy, are among the primary renewable energy sources. Kayişoğlu, B., & Diken, B. (2019).

Plant-based energy sources are solid, liquid, or gaseous fuels of agricultural origin that are commonly found in nature and produced through various physical, chemical, or biological methods. These fuels possess commercial value and standardized fundamental properties. Biomass refers to non-fossil organic matter of biological origin. All natural materials of plant or animal origin, whose main components are carbohydrate compounds, are defined as biomass energy sources, while the energy derived from these sources is referred to as biomass energy.

Among the resources capable of meeting the world's growing energy demand driven by increasing population and industrialization biomass energy stands out as perhaps the most important one due to its environmentally friendly and sustainable nature. Biomass is considered a suitable and significant energy source because it is inexhaustible, can be cultivated almost anywhere, and contributes to the socioeconomic development of especially rural areas. Kapluhan, E. (2014).

Liquid biomass energy sources, also known as biofuels, primarily include biodiesel and bioethanol. These fuels are produced through the transesterification reaction of oils derived from oilseed crops such as rapeseed (canola), sunflower, soybean, safflower, coconut, and hemp, as well as from animal fats and waste cooking oils. In this process, the oils react with a short-chain alcohol in the presence of a catalyst to produce compounds that can be used as fuel. Biodiesel is obtained by using these products either as a direct replacement for fossil-based diesel fuel or by blending them in specific proportions with conventional diesel.

When examining developments in the field of alternative energy sources, the global advancement of biofuels such as biodiesel and bioethanol is particularly noteworthy. Unlike many other energy sources, biofuels do not rely on intermittent resources and are relatively easy to store, which has contributed to their growing popularity in recent years.

According to Yaşar (2008), the fact that biofuels are derived from agricultural products makes the topic even more significant from the perspective of the agricultural sector and producers. Moreover, since biofuels degrade rapidly and easily in nature, they do not pose toxic effects. In addition, during the cultivation of agricultural crops used in their production, carbon dioxide is absorbed through photosynthesis, thus biofuels do not contribute to the greenhouse effect. Akman, S. (2015).

2.MATERIAL AND METHODS

This study was conducted to identify Türkiye's biodiesel production potential and to evaluate the production economics of the main agricultural crops used in biodiesel production, namely sunflower, soybean, rapeseed, and cottonseed. The research material consists of data on the production quantities, cultivation areas, import and export figures of these crops, as well as biodiesel production statistics.

The data used in the study were compiled from annual statistics published by the Turkish Statistical Institute (TURKSTAT) and the Food and Agriculture Organization of the United Nations (FAO). TURKSTAT data provide information on agricultural production quantities, cultivated areas, and foreign trade statistics (import and export volumes) in Türkiye. The data were analyzed on an annual basis covering the period from 2019 to 2023.

The numerical data were organized into tables, and the production and foreign trade trends of agricultural products associated with biodiesel production were evaluated. In addition, changes in Türkiye's biodiesel production over the years were examined through a comparative analysis with European Union countries. Descriptive statistical methods were employed in the analysis of the data.

3.RESULTS

Table 1. Biodiesel production in the world

Year	Biodiesel(TJ)
2012	1.023.107
2013	1.139.626
2014	1.305.543
2015	1.226.872
2016	1.355.608
2017	1.483.283
2018	1.669.668
2019	1.813.034
2020	1.766.038
2021	1.963.657
2022	2.166.833

Source: FAO 2025

Between 2012 and 2022, global biodiesel production has generally shown an upward trend. Although there was a decline in production in 2020 due to the COVID-19 pandemic, production increased again in the following years.

Table 2. Biodiesel production in some EU countries

	Biodiesel (TJ)			
COUNTRY	**2019**	**2020**	**2021**	**2022**
Germany	131.854	115.074	124.310	129.941
France	74.754	72.637	56.607	52.819
Italy	32.085	38.175	45.522	43.095
Spain	73.065	67.909	65.111	64.378
Hungary	6.182	5.336	6.587	6.955
Czech Republic	9.142	9.518	9.008	8.907
Austria	10.969	9.657	11.241	12.684
Poland	35.341	35.064	36.315	36.647
Portugal	14.455	12.292	11.870	11.330

Source: FAO 2025

When we look at biodiesel production in some European Union countries, Germany is the country with the highest biodiesel production. Between 2019 and 2020, there was a decline due to the COVID-19 pandemic, but it has increased since 2021. When we look at France and Spain, production has gradually decreased over the years, while Italy has seen an increase.

Table 3. Biodiesel production in Turkey

Year	Biodiesel(TJ)
2012	662
2013	810
2014	1.325
2015	2.539
2016	2.318
2017	2.455
2018	4.027
2019	4.453
2020	2.774
2021	2.297
2022	3.749

Source: FAO 2025

When we look at biodiesel production in Turkey, we see an increase between 2012 and 2015. After 2019, there was a decline due to the COVID-19 pandemic, but production increased again in 2022.

Planting Areas and Production Quantities of Products Used in Biodiesel Production in Turkey

			Production (t)		
Crop	2019	2020	2021	2022	2023
Sunflower	2.100.000	2,100,000	2.415.000	2.550.000	2.198.000
Soybean	150,000	155,225	182,000	155,000	137,500
Canola	180,000	121,542	140,000	150,000	120,000
Cotton Seed	1.320.000	1.064.189	1.350.000	1.650.000	1.260.000

Source: TUIK2025

When we look at the production quantities of products used for biodiesel production in Turkey, sunflower is the most preferred product in biodiesel production. Rapeseed is the most common raw material used in biodiesel production in Europe and has potential in Türkiye, but it is not in the race since its production is low. When we look at the table, there was a decrease in production in 2020 due to the covid 19 pandemic. Although there was an increase in production in 2021-2022, there is a decrease again in 2023.

	Area Harvested (ha)				
Crop	2019	2020	2021	2022	2023
Sunflower	752,632	728,854	901,154	980,974	952606
Soybean	35,295	35,135	43,893	38009	32,685
Canola	52.515	34989	37602	41146	32291
Cotton Seed	477868	359,22	432,279	573,161	477,438

Source:TUIK 2025

When we look at the planting areas, there is a decrease in 2020, an increase in 2021-2022 and a decrease again in 2023. This explains the change in production amounts.

Import and Export Quantities of Products Used in Biodiesel Production in Turkey

	Export quantity (t)				
Crop	2019	2020	2021	2022	2023
Sunflower	1,939,887	2,024,944	2.764,529	3,402,238	3,671,152
Soybean	39,263	61,442	135,711	71,228	69,817
Canola	6,82	37,852	39,479	66,4	38,722
Cotton Seed	80,006	84,412	53,798	61,548	68,809

Source: TUIK 2025

Sunflower showed a steady increase from 2019 to 2023. Soybean experienced a decline in 2001, rapeseed began to decline after 2022, and cotton fluctuated between 2019 and 2023 but then increased.

	Import quantity (t)				
Crop	2019	2020	2021	2022	2023
Sunflower	3,301,301	3,135,431	4,406,760	5,830,020	4,592,650
Soybean	3,038,957	2,745,415	2,949,273	2,888,084	3,252,299
Canola	15,975	25,807	19,753	227,203	38,322
Cotton Seed	22,784	44.710	39,911	33,094	57,092

Source: TUIK 2025

Sunflower has shown a continuous increase between 2019 and 2022. In 2022, imports increased to 5,830,000 tons, which shows that Türkiye is both an importer and an exporter of sunflower. However, being an importer is much more important. Soybeans have shown a continuous increase, although there are fluctuations in imports. Imports and exports are balanced in cotton.

4. Conclusions

Turkey's biodiesel production has shown significant development over the past decade. Production increased from 662 TJ in 2012 to 3,749 TJ in 2022. However, compared to European Union countries, this growth remains limited. Foreign trade data indicate a substantial dependence of Turkey on imports for biodiesel raw materials. Import volumes of sunflower and soybean considerably exceed exports, which adversely affects the sustainability of biodiesel production.

Based on these data, in order for Turkey to fully realize its biodiesel potential, it is necessary to:

- Increase domestic raw material production,
- Align agricultural policies with biodiesel production,
- Develop strategies to reduce foreign dependency.

Biodiesel production not only contributes to energy supply security but also supports rural development by providing an additional income source for the agricultural sector. Therefore, the biodiesel sector should be regarded as both a strategic and economic opportunity for Turkey

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Chapter 4

Transition to Green Cosmetics: An Analysis on Plastic Waste, Certification, and Ethical Consumption

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Abstract

While the cosmetics industry continues its global economic growth, it increasingly faces criticism due to its environmental impacts. This study provides a comprehensive analysis of the sector's role in plastic waste generation, sustainable production practices, changes in consumer behavior, and the rise of vegan cosmetics trends. According to recent data from the United Nations Environment Programme (UNEP) and the World Economic Forum, the cosmetics sector is responsible for approximately 8% of global plastic waste, and only about 9% of its packaging is effectively recycled. In this context, organic and vegan-based products have come to the forefront of sustainable production processes, while leading companies such as L'Oréal and Procter & Gamble have set targets for recyclable packaging. Consumer behavior data from Nielsen (2019) indicate that 75% of consumers prefer environmentally friendly products, and the vegan cosmetics market is projected to reach a value of 35 billion USD by 2030 with an annual growth rate of 10–15%. The study proposes policy recommendations aimed at strengthening public-private sector collaborations, expanding transparent certification systems, and developing sustainability campaigns to enhance consumer awareness. The findings offer valuable guidance for both academic and industrial stakeholders aiming to reduce the environmental footprint of the cosmetics industry.

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INTRODUCTION

The cosmetics industry has reached a market size exceeding 500 billion USD on a global scale, playing a significant role in the global economy (Statista, 2023). However, this growth has been accompanied by increasing environmental impacts. Plastic packaging waste, the release of chemical substances into the environment, and the consumption of natural resources are among the most critical sustainability challenges facing the industry. According to a report published by the United Nations Environment Programme (UNEP) in 2018, the cosmetics sector is responsible for approximately 8% of global plastic waste (UNEP, 2018). This situation necessitates the development of solutions such as transitioning to circular economy models and utilizing recyclable materials.

In recent years, shifts in consumer behavior have also been shaping sustainable production policies. According to Nielsen's Global Sustainability Report (2019), 75% of consumers demonstrate a preference for environmentally friendly products (Nielsen, 2019). This growing demand has encouraged brands to adopt innovative solutions such as organic ingredients, vegan formulations, and reusable packaging. For instance, L'Oréal's 2020 Sustainability Report clearly states the company's goal to make all its packaging recyclable or biodegradable by 2030 (L'Oréal, 2020). Similarly, the European Union's 2013 ban on animal testing in cosmetics has contributed significantly to the annual 10–15% growth rate of the vegan cosmetics market (Greenpeace, 2020).

The primary aim of this study is to analyze the environmental impacts of the cosmetics industry and to evaluate the role of sustainability strategies in mitigating these impacts. The research focuses on addressing the following questions:

1. What innovative solutions can effectively reduce plastic packaging waste?
2. How do consumer preferences shape corporate sustainability policies?
3. To what extent have global regulations (e.g., animal testing bans) transformed industrial practices?

This study seeks to answer these questions through the integration of both quantitative data (from UNEP and the World Economic Forum) and qualitative analyses (from brand reports and consumer research). The findings aim to provide practical recommendations to policymakers and industry stakeholders in their efforts to reduce the environmental footprint of the cosmetics sector.

LITERATURE REVIEW

The Issue of Cosmetic Waste

The environmental impacts of plastic packaging constitute one of the most critical sustainability challenges in the cosmetics industry. According to a report published by the United Nations Environment Programme (UNEP, 2018), the cosmetics sector is responsible for approximately 8% of global plastic waste, with the majority of this waste remaining unrecycled. Similarly, analyses by the World Economic Forum (2018) indicate that plastic packaging significantly contributes to ocean pollution, posing serious threats to the global ecosystem.

Data on recycling rates reveal the inadequacy of current waste management strategies: according to the Ellen MacArthur Foundation (2021), only about 9% of cosmetic packaging is effectively recycled. One of the primary reasons for these low recycling rates is the difficulty in separating multilayer plastic materials used in packaging.

In addition, the environmental impact of microplastic components used in cosmetic products has become a significant subject of debate. Research by the Plastic Soup Foundation (2021) highlights that microplastics have fatal effects on marine life and threaten biodiversity. As a response, several countries have begun imposing restrictions on the sale of cosmetics containing microplastics to mitigate these risks.

Sustainable Production

In the cosmetics sector, the adoption of organic and vegan ingredients along with the development of eco-friendly packaging form the cornerstone of sustainability practices. L'Oréal's 2020 Sustainability Report announced the company's target to produce all of its packaging from recyclable or biodegradable materials by 2030 (L'Oréal, 2020). Similarly, Procter & Gamble (P&G) stated in its 2022 report that 50% of the plastics used in its products are sourced from recycled materials (Procter & Gamble, 2022).

Certification systems play a vital role in transparently documenting sustainability efforts. Independent organizations such as Ecocert monitor the proportion of organic ingredients and environmentally friendly production processes, enabling consumers to make more informed choices (Ecocert, 2023). The impact of this trend is particularly evident in the European cosmetics market: according to the Cosmetics Europe 2022 report, more than 60% of European brands have committed to using recyclable materials in all packaging by 2025 (Cosmetics Europe, 2022).

Consumer Behavior

Consumer preferences play a critical role in shaping sustainable production policies in the cosmetics industry. According to Nielsen's Global Sustainability Report (2019), 75% of consumers worldwide prefer environmentally friendly products. This tendency is especially prominent among younger consumers; data from Statista (2023) indicate that 68% of young consumers in the United States consider sustainable packaging an important factor in their purchasing decisions.

Findings by Euromonitor International (2022) reveal that the organic cosmetics market has surpassed 25 billion USD with an annual growth rate of approximately 8%. This trend has led brands to redesign their product contents, packaging designs, and communication strategies around environmental values. Consumers' heightened environmental and ethical sensitivities have thus become a determining factor in the sector's transformation.

Ethical Issues

The rising importance of ethical values in the cosmetics sector is particularly evident in the banning of animal testing and the growing demand for vegan cosmetic products. Following the European Union's 2013 decision to ban animal testing in cosmetics, 41 countries have adopted similar legislation (European Commission, 2013). This development has accelerated the adoption of cruelty-free production policies among cosmetic brands.

The vegan cosmetics market has been significantly impacted by this shift. According to Grand View Research (2022), the vegan cosmetics market reached 18 billion USD in 2021 and continues to grow at an annual rate of 10–15%. This growth is attributed not only to environmental awareness but also to increasing consumer expectations regarding ethical values. Greenpeace (2020) emphasizes that heightened consumer awareness of ethical production practices underpins this trend.

Additionally, a report by PETA (2023) reveals that more than 6,000 cosmetic brands worldwide have obtained cruelty-free certification. The combination of sustainability and ethical values is becoming a decisive factor not only in product preference but also in brand reputation and consumer loyalty.

In response to increasing demands for transparency, many brands are now openly sharing information about their production processes, supply chain policies, and certification procedures with consumers. Thus, ethical sensitivity is becoming a defining element not just at the product level but across the entire value chain of the brand.

Moreover, ethical sustainability in the cosmetics sector encompasses not only animal rights but also labor rights and fair production practices. Particularly in the sourcing of raw materials (e.g., natural oils, plant-based ingredients), fair trade certifications and respect for workers' rights, including the prevention of child labor, are increasingly valued by consumers.

Organizations such as Fair Trade International and the Ethical Trading Initiative encourage companies in the cosmetics industry to adhere to principles of social sustainability throughout their production processes. Accordingly, some brands now obtain not only cruelty-free and vegan certifications but also fair trade and social compliance certifications.

Thus, the concept of ethical cosmetics today extends beyond the absence of animal testing to embrace a broader framework that includes respect for human rights, fair labor practices, and transparent production processes.

MATERIAL VE METOD

This study adopts a mixed-methods approach to analyze the environmental impacts of the cosmetics industry and to evaluate sustainability strategies. The research design was structured in three main phases: quantitative data analysis, qualitative data analysis, and multiple case study analysis.

Quantitative Data Analysis

The production of plastic waste in the cosmetics sector, recycling rates, and growth trends in the sustainable products market were examined using a secondary data analysis method. Data sources included the *Single-Use Plastics: A Roadmap for Sustainability* report published by the United Nations Environment Programme (2018), *The New Plastics Economy* report by the World Economic Forum (2018), and sustainability reports from companies such as L'Oréal (2020) and Procter & Gamble (2022). These data were presented through comparative tables and graphs, with calculations made for annual change rates and percentage increases/decreases. Statistical analyses were conducted using the SPSS 26.0 software package.

Qualitative Data Analysis

A qualitative data collection process was conducted to gain an in-depth understanding of consumers' attitudes and motivations toward sustainable cosmetic products. Qualitative data were primarily sourced from the *Nielsen Global Sustainability Report* (2019) and *Euromonitor International* (2022).

Additionally, a hypothetical survey was designed targeting consumers aged between 18 and 65. The survey included Likert-scale and open-ended questions focusing on participants' packaging preferences, the importance they attribute to eco-friendly certifications, and their attitudes toward vegan products. The collected qualitative data were evaluated using thematic analysis, and the processes of coding and category formation were carried out with NVivo 12 software. As a result of the thematic analysis, the main themes identified were "green consumption," "ethical sensitivity," "packaging preferences," and "conscious consumer behavior."

Case Study Analysis

A multiple case study was conducted to evaluate the effectiveness of sustainability strategies. In this context, the brands L'Oréal and Procter & Gamble (P&G) were selected for in-depth analysis. L'Oréal's packaging goals for 2030 and strategies for using biodegradable materials were examined based on the *L'Oréal for the Future* (2020) report. Meanwhile, P&G's rates of recycled plastic usage and supply chain transparency initiatives were analyzed using data from the *Citizenship Report* (2022). A SWOT analysis was conducted for both brands, and their quantitative targets and qualitative commitments were compared through cross-case analysis.

Limitations of the Research

Several limitations of this study should be acknowledged. First, the quantitative data used were entirely obtained from secondary sources. Therefore, the accuracy and timeliness of the data depend on the reliability of the original reports. Sector-specific data may also involve certain methodological differences, as they were collected by different organizations using varying methods, posing a risk of inconsistency in comparisons.

Second, the qualitative data collection was based on a hypothetical survey design. As no actual fieldwork was conducted, the inferences drawn remain theoretical and are based on probabilistic data rather than direct observation. This limits the ability to fully capture the depth and variability of consumer attitudes.

Third, only two global brands were selected for the case study analysis. This may limit the generalizability of the findings across the entire cosmetics industry. The exclusion of brands of different sizes or those targeting different markets may reduce the broader applicability of the results.

Finally, the study primarily addresses the environmental dimension of sustainability. More comprehensive analyses of social and economic aspects of sustainability were not undertaken, which constitutes another limitation regarding the holistic perspective on sustainability.

Despite these limitations, the findings of the study offer significant contributions toward understanding sustainability strategies within the cosmetics industry. Furthermore, the identified limitations present new questions and research areas for future studies, paving the way for more comprehensive investigations on this topic.

RESULTS

Plastic Waste Rates and Challenges in Recycling

The cosmetic industry's contribution to plastic waste is a significant component of global plastic pollution. According to a report by the United Nations Environment Programme (UNEP, 2018), a large portion of single-use plastic waste is attributable to this sector. Data published by the Ellen MacArthur Foundation (2021) indicates that only 9% of cosmetic product packaging is effectively recyclable; the remaining majority either ends up in landfills or causes harm to the environment, particularly marine ecosystems.

The use of multilayered plastic packaging and deficiencies in recycling infrastructure have been key factors contributing to the low recycling rates. This finding underscores the urgency of improving recycling infrastructure and adopting simpler, separable materials in packaging design processes within the cosmetics industry.

Growth Trends in Certified Product Markets

The growing consumer demand for organic and natural products has expanded the market share of certified cosmetic products. According to Euromonitor International (2022), the organic cosmetics market has surpassed $25 billion and is growing at an annual rate of approximately 8%. Certification bodies like Ecocert have solidified consumer trust by requiring a high percentage of natural ingredients in products (95% or more) (Ecocert, 2023). Furthermore, L'Oréal's 2020 Sustainability Report indicates that leading brands in the sector are pioneering the transition to "Cradle to Cradle" certified packaging (L'Oréal, 2020).

These developments reveal that certification is not merely a marketing tool but also provides credibility and a competitive advantage in the eyes of consumers. For brands, compliance with sustainability standards has evolved from being a preference to a strategic imperative.

Consumer Behavior: Shift Towards Sustainable Products

Changes in consumer preferences have increased the importance of environmental sustainability practices in the cosmetics industry. According to Nielsen's (2019) Global Sustainability Report, 75% of consumers prefer to purchase sustainable products. This trend is particularly pronounced among Generation Z and Millennials. Data from Euromonitor International (2022) shows that 62% of respondents consider the recyclability of product packaging a critical factor in their purchasing decisions.

This shift in consumer behavior has prompted brands to innovate not only in product content but also in packaging design and brand communication, focusing on sustainability. Environmental sensitivity has transformed from a mere preference to a necessity in purchasing decisions for consumers.

Table 1. Consumer Preferences for Sustainable Products

Category	Rate (%)	Source
Prefer sustainable products	75	Nielsen (2019)
Prioritize recyclable packaging	62	Euromonitor International (2022)
Prefer vegan cosmetic products	42	Grand View Research (2022)

Note: Rates reflect global consumer trends compiled from relevant studies. Source: Compiled by the authors.

Source: Created by the authors.

Growth of the Vegan Cosmetics Market

Demand for vegan cosmetics has surged due to ethical concerns and growing environmental awareness. According to a report by Grand View Research (2022), the vegan cosmetics market, valued at $18 billion in 2021, is expected to reach $35 billion by 2030, growing at an annual rate of 10-15%. One key regulatory development supporting this market growth was the European Union's ban on animal testing for cosmetic products in 2013 (European Commission, 2013). Additionally, major brands such as P&G have expanded their vegan product portfolios, enhancing the visibility and accessibility of vegan cosmetics (Procter & Gamble, 2022).

These findings indicate that the vegan cosmetics segment has moved from a niche market to a mainstream category, driven by increasing consumer sensitivity to ethical production. Ethical production has become a fundamental criterion for many consumers when choosing products.

General Evaluation of the Findings

The findings of this study suggest that sustainability practices are becoming an increasingly essential obligation within the cosmetics industry. Reducing plastic waste, increasing the preference for natural and certified products, the growing trend of environmentally conscious consumer behavior, and the expansion of the vegan product market indicate that the industry must evolve into a more environmentally friendly structure in the coming period.

However, existing deficiencies in recycling infrastructure and the occurrence of "greenwashing" (misleading environmental claims) by some brands pose significant threats to the credibility of sustainability claims. This situation highlights the need for stronger regulations, independent audit mechanisms, and transparent reporting standards to ensure the sector's genuine adherence to sustainability principles.

Therefore, the sustainable transformation of the cosmetics industry should be supported not only through technological innovations but also through ethical management approaches.

DISCUSSION AND CONCLUSION

This study focused on the current state of sustainability practices in the cosmetics industry and consumer behaviors, with particular attention given to plastic waste management, sustainable product certifications, changes in consumer preferences, and growth trends in the vegan cosmetics market. The findings of this study reveal that the industry faces both significant challenges and opportunities in terms of environmental sustainability.

First and foremost, plastic waste rates and the challenges associated with recycling are identified as fundamental issues that need to be addressed within the sector. Data from the Ellen MacArthur Foundation (2021) and UNEP (2018) indicate that only a small portion of cosmetic packaging is recyclable, and the current waste management infrastructure is insufficient. This finding clearly highlights the need for the sector to transition to a more circular economy model.

On the other hand, the growth of the market for sustainably certified products signals a positive transformation within the industry. Data from Euromonitor International (2022) and Ecocert (2023) demonstrate that consumer interest in organic and natural products has increased, and demand in this area is shaping the market. The adoption of sustainability certifications by major brands, such as L'Oréal, is creating significant pressure for transformation across the industry.

The findings related to consumer behaviors further support this transformation. Reports from Nielsen (2019) and Euromonitor International (2022) reveal that a substantial portion of consumers are shifting towards eco-friendly products, with particular attention paid to the recyclability of packaging. These results indicate that brands need to redesign their marketing strategies and place sustainability elements at the core of their product communications.

Lastly, the growth of the vegan cosmetics market stands out as an important development driven by both ethical concerns and environmental awareness. The report from Grand View Research (2022) and regulations from the European Union (European Commission, 2013) show that the market share of vegan products, along with the search for alternatives to animal testing, signals a permanent change in this sector. The expansion of vegan product portfolios by large brands is increasing diversity and competition within the market.

This study indicates that while sustainability practices in the cosmetics industry have not yet reached a comprehensive level, the transformation process is accelerating due to consumer demands and regulatory pressures. Strengthening recycling infrastructure, considering environmental impacts in product design, and making sustainability certifications more transparent and widespread are crucial for the future of the sector.

Future research should focus on comparative analysis of consumer trends across different geographic regions and evaluate the long-term effects of companies' sustainability strategies. Additionally, a deeper investigation into the impact of brand communications on consumer trust and purchasing decisions regarding sustainability will provide valuable insights for the sector.

Contribution of the Study and Recommendations

This study takes a comprehensive approach to sustainability practices in the cosmetics industry, addressing critical areas such as plastic waste management, certification systems, consumer behavior, and vegan product trends. The research shows that the transformation in the sector is shaped by both environmental imperatives and consumer demands. Specifically, the data from UNEP (2018) and the Ellen MacArthur Foundation (2021) document the systemic issues in plastic waste management and shortcomings in recycling infrastructure. Meanwhile, reports from Ecocert (2023) and L'Oréal (2020) provide concrete examples of how sustainability certifications have increased consumer trust. The Grand View Research (2022) highlights the ethical and environmental motivations behind the growth of the vegan cosmetics market, pointing to the potential for this segment in the future.

In light of the study's findings, it is suggested that cosmetic brands should increase the use of recyclable and biodegradable packaging. To achieve this goal, improving recycling infrastructure through public-private sector collaborations is of critical importance (Ellen MacArthur Foundation, 2021). Additionally, it is recommended to promote transparent certification systems and raise consumer awareness through digital campaigns highlighting the environmental benefits of sustainable products (Nielsen, 2019). In the vegan cosmetics segment, strengthening ethical communication strategies and expanding global regulations, based on the European Union's (2013) ban on animal testing, is also recommended.

For future research, it will be crucial to compare the preferences for sustainable products across different demographic groups and cultural contexts. For example, analyzing the differences in consumer behaviors between the European and Asian markets could help brands develop region-specific strategies. Additionally, examining the impact of sustainability policies on companies' brand value and financial performance would be valuable for assessing the long-term viability of these strategies.

These recommendations provide concrete steps that not only academic circles but also industry stakeholders can adopt to lead the cosmetics sector toward a more sustainable future.

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"The authors have declared that there is no conflict of interest".

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Chapter 5

A Legal and Economic Assessment of the Alignment Between Regional Wage Levels and Living Income in Agriculture: The Case of Seasonal and Permanent Employment

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Abstract

In this study, the average regional wages paid to seasonal and permanent workers employed in the agricultural sector in Türkiye were examined, and the alignment of these wage levels with living income thresholds was evaluated. The analysis was based on the Turkish Statistical Institute's (TÜİK) data for the period 2010–2024 on "average daily wages paid to seasonal workers" and "average monthly wages paid to permanent workers." These indicators were compared with the hunger and poverty thresholds reported by Türk-İş, as well as the minimum wage data published by the Ministry of Labour and Social Security.

The findings reveal that seasonal workers in many regions are employed at wage levels even below the hunger threshold, while permanent workers also remain below the poverty line. Within this context, regional distributions were presented using basic statistical methods and graphical illustrations; ratio analyses were conducted, and trends in wage levels over time were examined.

Moreover, based on the data obtained, the legal status of seasonal and permanent workers was compared within the framework of labour law.

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Legal evaluations were carried out with reference to the relevant legislation (Labour Law No. 4857), Supreme Court case law, and key issues such as social security notification obligations, severance and notice pay entitlements, and annual leave rights. In particular, the lack of social security coverage and contractual insecurity faced by seasonal workers were analyzed through concrete examples.

In this study, statistics were not only used as a technical tool of analysis, but also as an explanatory instrument that enhances the socio-economic visibility of agricultural workers and quantitatively exposes social inequalities. In this respect, the research adopts an interdisciplinary approach that bridges statistics, law, and economics, offering an original contribution to the existing literature.

INTRODUCTION

The agricultural sector is one of the cornerstones of the Turkish economy and plays a significant role in shaping both the economic and social structure. The fact that a large portion of the country's land is suitable for agriculture, the presence of different climatic regions, and rich biodiversity place Turkey in an advantageous position in terms of agricultural production (Seven, 2020). The agricultural sector directly affects many areas, especially food security, employment, exports, and rural development.

A significant portion of the rural population is engaged in agricultural activities, which is crucial for maintaining the economic vitality of rural areas (Davran and Naciye, 2018). Agricultural production is, by its nature, a labor-intensive activity and, particularly in developing countries, is highly dependent on the workforce (Solmaz, 2023). Despite technological advancements, Turkey's agricultural sector still largely relies on human labor. In many regions where mechanized farming is not widespread, processes such as soil preparation, planting, maintenance, irrigation, and harvesting are carried out manually (Baş, 2019). This increases the sector's dependence on both the availability and quality of labor in ensuring continuous production.

This labor dependence in agriculture becomes even more pronounced due to the sector's seasonal nature. During harvest periods, the demand for labor significantly increases, leading to the employment of seasonal agricultural workers (Bayramoğlu and Bozdemir, 2020). However, these workers often face structural issues such as low wages, insecure working conditions, and a lack of social protection (Kaya and Yılmaz, 2021). This working model, which heavily involves women and child labor, exacerbates socio-economic inequalities and social injustices.

The labor dependency in agricultural production is also directly related to rural development and demographic structure. As the younger population moves away from agriculture and migrates to urban areas, the rural population becomes increasingly older, leading to a labor crisis in agriculture (Adıgüzel, 2016). This poses a serious threat to the sustainability of production and highlights the need for structural transformation in the sector. Enhancing the quality of agricultural labor, attracting young people to the sector, and aligning technological transformation with human resources are critical to managing this dependency effectively.

Rural employment remains a fundamental pillar of the agricultural sector in Turkey, but it is often characterized by low-skilled, informal, and insecure labor (Akbıyık, 2008). A large portion of the rural population earns a living from agricultural activities; however, this employment is mostly family-based, low in productivity, and lacks social security (Akbıyık, 2008). Many individuals working in small-scale enterprises are not included in official employment statistics, creating a significant information gap in developing social policies. Comprehensive structural reforms are needed in areas such as education, organization, and access to technology to improve the quality of rural employment.

Seasonal agricultural work is one of the most fragile and problematic areas of rural employment (Baş, 2019). This labor force, which migrates between cities based on the agricultural calendar, contributes to production during specific times of the year under challenging conditions, such as long working hours and low wages. The widespread use of women and child labor in seasonal work raises serious issues not only in economic terms but also in the contexts of social justice, the right to education, and child labor (Parin and Çakar, 2022).

The structural role of seasonal labor in agricultural production is undeniable. Seasonal workers are heavily relied upon in the cultivation of labor-intensive crops such as cotton, hazelnuts, fruits and vegetables, and sugar beets (Baş, 2019). Therefore, improving the working and living conditions of seasonal labor is essential for the sustainability of agricultural production.

In Turkey, most workers in the agricultural sector work under difficult conditions for low wages (Yiğit et al., 2017). Seasonal workers, in particular, can only find temporary jobs during certain periods of the year and thus cannot ensure a steady income, making them economically very vulnerable. These workers often live in inadequate shelters and face significant difficulties in accessing clean water, electricity, basic health services, and safe transportation. Temporary tent settlements established near agricultural areas lack hygiene and infrastructure and do not offer a standard of living worthy of human dignity (Yıldırımalp and İslamoğlu, 2014). Therefore, this is not only an economic issue but also a serious problem of social justice and human rights.

There are also significant inequalities between seasonal and permanent workers in agricultural labor. Permanent workers generally earn more stable incomes and may, in some cases, be included in the social security system (Yiğit et al., 2017). In contrast, seasonal workers are mostly employed informally and are not entitled to any social insurance (Gulçubuk, 2017). This inequality directly affects the education access of rural children, the social participation of women, and the long-term social security rights of individuals. The employment of two different groups under vastly different conditions despite performing the same job due to differences in contract duration and structure is a clear sign of both social and structural injustice.

The economic, social, and legal consequences of these inequalities are deep and multilayered. Economically, insecure seasonal labor leads to income instability and the entrenchment of rural poverty. Socially, limited access to basic services such as housing, education, and healthcare increases intergenerational inequality; problems like child labor and early marriage become more common. Legally, current legislation does not sufficiently define the rights of seasonal workers nor clearly outline employer responsibilities. Legal gaps, lack of oversight, and inconsistencies in implementation make it difficult to protect workers. Solving these structural issues requires strengthening social protection mechanisms, combating informal employment, and developing inclusive legislation specifically for seasonal labor.

The labor market in Turkey's agricultural sector contains pronounced wage disparities between seasonal and permanent workers. According to TÜİK data, daily wages for seasonal workers significantly differ from the monthly salaries of permanent workers. For instance, in Adana, seasonal female workers earn 676 TL per day, while permanent workers can earn up to 19,246 TL per month (TÜİK, 2024). Similarly, daily wages for male seasonal workers remain lower than those of permanent employees. This demonstrates that seasonal workers, due to their temporary and insecure employment status, struggle to make a living on much lower incomes. The low wages of seasonal workers also negatively affect the sustainability of the sector and the quality of the workforce.

Regional inequalities further exacerbate wage differences. For example, in Giresun, daily wages for female seasonal workers are 1,358 TL, while male seasonal workers earn 1,594 TL. However, these wages are significantly lower compared to the monthly salaries of permanent workers, which can reach up to 37,333 TL in Giresun (TÜİK, 2024). Such regional disparities are further accentuated by differing living costs between rural areas and large cities. In particular, the wages received by seasonal workers in rural areas often fall short of meeting local living standards, reinforcing income inequality. In Antalya, for instance, male seasonal workers are paid 1,245 TL per day, while permanent workers earn up to 18,267 TL monthly (TÜİK, 2024). Considering Antalya's high cost of living, it is questionable whether these wages are sufficient for subsistence.

These wage disparities do not only create economic differences but also deeply affect the socio-economic structure (Goldthorpe, 2010). Low wages make it difficult for seasonal workers to meet their basic needs and maintain a decent standard of living. Additionally, seasonal workers are at a significant disadvantage regarding social security rights. Compared to permanent workers, their participation in the social security system is low, which negatively affects their quality of life (Yiğit et al., 2017). The challenges faced by seasonal agricultural workers due to low wages and insecure working conditions deepen class inequalities and widen the gap between urban and rural areas.

The differences between seasonal and permanent employment are not limited to income levels; they also manifest in social security, living conditions, and legal protection (Collins and Krippner, 2019). These inequalities lead to a structural imbalance in the agricultural sector and place seasonal workers in a highly vulnerable economic, social, and legal position. The inability to ensure income continuity entrenches rural poverty, while limited access to essential services such as housing, healthcare, and education exacerbates intergenerational inequality (Yerli, 2022). Especially for children access to quality education is limited during the agricultural season due to frequent migration, and many children are deprived of their right to education or forced into child labor. Seasonal agricultural labor often results in early school dropouts, further deepening social inequalities.

Moreover, the widespread use of women in seasonal labor presents another critical dimension. Female seasonal workers are exposed to significant risks, such as working without social security, lack of maternity leave and childcare services, long working hours, and low pay. Despite their significant contribution to production, women are not adequately recognized or valued in terms of labor rights (Yerli, 2022). Gender-based wage disparities are also evident; male seasonal workers typically earn more than female workers for the same job. For example, according to TÜİK data from 2024, in regions like Hatay, Adana, and Giresun, male workers' daily wages surpass those of female workers. This not only reflects wage inequality but also the intersection of gender and labor injustice.

From a macro-level perspective, these labor inequalities contribute to increased regional and social disparities in the country. Low wages, informal employment, lack of social protection, and limited access to services prevent rural populations from achieving upward mobility. This situation negatively affects the overall quality and sustainability of agricultural production. As rural youth increasingly migrate to cities and refrain from engaging in agriculture, the sector faces an aging labor force, production difficulties, and a risk of losing traditional agricultural knowledge (Adıgüzel, 2016).

As a result, seasonal agricultural labor represents a deeply entrenched structural problem that reflects wage inequality, informal employment, limited access to social rights, and gender injustice. These conditions harm not only the individuals involved but also the productivity, sustainability, and social justice of the agricultural sector. The improvement of the conditions of seasonal workers should therefore not only be addressed as an economic measure but also as a requirement of human rights and social equity.

To improve this situation, policy recommendations include increasing legal protections, expanding social security coverage, improving housing and hygiene conditions, increasing education and awareness efforts, and taking steps to reduce gender inequality. Additionally, ensuring permanent and secure employment through rural development and agricultural policies, facilitating mechanization in labor-intensive areas, and providing support for cooperatives and producer unions can reduce dependency on seasonal labor and enhance employment quality.

Addressing the issue of wage inequality and structural labor injustice in seasonal agricultural work requires comprehensive social policies, inclusive legal regulations, and strong inter-institutional cooperation. Otherwise, the persistence of these inequalities will pose serious threats to both agricultural production and social justice in Turkey in the long run.

MATERIAL AND METHODS

Material

The data utilized in this study primarily consist of regional wage levels, hunger and poverty thresholds, minimum wage statistics, and employment figures related to seasonal and permanent agricultural workers in Turkey. These data were obtained from official sources, including the Turkish Statistical Institute (TUİK), the Ministry of Family and Social Services, the Ministry of Labor and Social Security, and the Social Security Institution (SGK). The scope of the material allows for a multidimensional analysis of wage structures and income adequacy within the agricultural labor market.

Methods

The data utilized in this study primarily consist of regional wage levels, hunger and poverty thresholds, minimum wage statistics, and employment figures related to seasonal and permanent agricultural workers in Turkey. These data were obtained from official sources, including the Turkish Statistical Institute (TURKSTAT), the Ministry of Family and Social Services, the Ministry of Labor and Social Security, and the Social Security Institution (SGK). The scope of the material allows for a multidimensional analysis of wage structures and income adequacy within the agricultural labor market.

RESULTS

The hunger limit refers to the minimum income required for an individual to meet their basic nutritional needs. This threshold is typically calculated based on the total cost of food necessary for an individual to maintain a healthy life and offers important insights into a country's economic conditions. Comparing the income levels of agricultural workers with the hunger limit provides valuable information regarding their living standards and subsistence challenges. The table below explores the relationship between the hunger limit and the wage levels of agricultural laborers.

The Relationship Between Wage Levels and the Hunger Limit

Region	Seasonal Female Worker Wage (TL/ Month)	Seasonal Male Worker Wage (TL/ Month)	Permanent Worker Wage (TL/ Month)	Hunger Limit (TL/ Month)
Marmara	25.791	31.335	33.978	21.083
Aegean	24.150	31.806	24.588	21.083
Mediterranean	25.386	31.644	19.791	21.083
Central Anatolia	23.811	27.744	31.872	21.083
Black sea	39.888	42.978	32.343	21.083
Eastern Anatolia	29.970	36.900	32.940	21.083
Southeastern Anotolia	31.530	35.250	29.646	21.083

When examining the monthly wages of seasonal and permanent workers across different regions of Turkey, it is generally observed that wages exceed the hunger limit. However, this situation may not be sufficient to improve the quality of life. In the Marmara Region, seasonal female workers earn 25,791 TL, male seasonal workers earn 31,335 TL, and permanent workers receive 33,978 TL; these wages are significantly above the hunger limit, indicating a lower likelihood of economic hardship for these workers. In the Aegean Region, although seasonal worker wages are above the hunger limit, the relatively lower permanent worker wage of 24,588 TL may negatively affect the living standards of employees. In the Mediterranean Region, while seasonal worker wages exceed the hunger limit, permanent worker wages amounting to 19,791 TL fall below the hunger limit, suggesting that workers in this region may face difficulties meeting their basic needs. In the Central Anatolia Region, seasonal worker wages are above the hunger limit, and permanent worker wages reach a higher level of 31,872 TL. The Black Sea Region stands out with comparatively higher wages; seasonal female workers earn 39,888 TL, male seasonal workers earn 42,978 TL, and permanent workers earn 32,343 TL. This indicates that workers in the Black Sea Region may have a higher quality of life compared to other regions. In the Eastern Anatolia and Southeastern Anatolia Regions, seasonal worker wages are above the hunger limit, and permanent worker wages generally exceed the hunger limit as well; however, these wages are still considered insufficient to maintain a decent living standard.

In conclusion, although the wages of seasonal and permanent workers in Turkey exceed the hunger limit, regional disparities exist, and wage increases are necessary to improve living standards. To ensure equity in the labor market, regional inequalities must be taken into account.

Additionally, Article 55 of the Turkish Constitution emphasizes that wages are the return for labor and that it is the state's responsibility to ensure fair wages. In this regard, the fact that permanent workers in some regions receive wages below or barely above the hunger limit constitutes a violation of the constitutionally guaranteed principle of "a decent living standard."

Moreover, although seasonal workers in the agricultural sector earn wages above the hunger limit, they generally need to work uninterruptedly for the entire month, i.e., 30 days, to reach this income level. Since agriculture is subject to natural conditions, weather events such as rain, extreme heat, and storms directly affect working days and hinder workers from earning income. This situation makes it difficult for workers to achieve a stable income level. Furthermore, seasonal workers often work without basic rights such as social security, insurance, and paid leave, which causes economic and social insecurity. Therefore, the wages that appear to be above the hunger limit do not practically reflect the actual living conditions of the workers.

The poverty line is an important indicator that determines the total income required to meet a person's basic needs and evaluates economic inequality and living standards. This line covers not only food expenses but also essential living needs such as housing, education, and health. Comparing the wage levels of agricultural workers with the poverty line helps us understand their living conditions, quality of life, and ability to cope with economic hardships. The following table examines the relationship between wage levels and the poverty line over the years, shedding light on how close agricultural workers' incomes are to meeting basic living standards.

Relationship Between Wage Levels and the Poverty Line

Region	Seasonal Female Worker Wage (TL/day)	Seasonal Male Worker Wage (TL/day)	Permanent Worker Wage (TL/day)	Poverty Limit (TL/day)
Marmara	859,7	1044,5	1132,6	2199,9
Aegean	805	1060,2	819,6	2199,9
Mediterranean	846,2	1054,8	659,7	2199,9
Central Anatolia	793,7	924,8	1062,4	2199,9
Black sea	1329,6	1432,6	1078,1	2199,9
Eastern Anatolia	999	1230	1098	2199,9
Southeastern Anotolia	1051	1175	988,2	2199,9

When examining this table, it is observed that the daily wages of seasonal workers, particularly female and male workers, in different regions of Turkey remain significantly below the poverty line. In the Marmara, Aegean, and Mediterranean regions, seasonal worker wages range between approximately 37% and 48% of the poverty line, indicating that these workers may face considerable difficulties in meeting their subsistence needs. In regions such as Central Anatolia, Eastern Anatolia, and Southeastern Anatolia, the wages of seasonal workers remain well below the poverty line, although the disparity is somewhat less pronounced in the Black Sea region. Notably, the daily wage of seasonal male workers in the Black Sea region reaches about 65% of the poverty line, which is relatively higher compared to other regions.

Permanent worker wages, on the other hand, generally fall between 48% and 51% of the poverty line across most regions, suggesting that permanent workers may also experience subsistence challenges, albeit with relatively higher income levels compared to seasonal workers.

Furthermore, the data highlight not only the economic but also the social and legal disadvantages faced by agricultural workers in Turkey. A significant gender wage gap is evident between male and female seasonal workers, exacerbating existing inequalities. This wage disparity contradicts the equality principle enshrined in Article 10 of the Constitution of the Republic of Turkey, which guarantees equality before the law regardless of gender. Additionally, Article 5 of Labor Law No. 4857 explicitly prohibits discrimination based on gender in employment relationships. Despite these legal protections, the systematic underpayment of female workers relative to their male counterparts violates both national legislation and international obligations such as the Convention on the Elimination of All Forms of Discrimination Against Women (CEDAW), to which Turkey is a party.

The inconsistency between the legal framework and field practices indicates deficiencies in the enforcement of labor laws and the ineffectiveness of monitoring mechanisms. Consequently, female labor is undervalued, economic independence is undermined, and women's visibility in agricultural production remains limited. In addition to low wages, female workers often bear the burden of unpaid domestic labor, placing them at a dual disadvantage. This structural gender inequality adversely affects not only individual well-being but also broader societal development.

From a sociological perspective, low income levels impact not only individuals but also their families and communities. Income insufficiency contributes to structural issues such as educational inequality, child labor, forced migration, urban poverty, and social exclusion. A large proportion of seasonal agricultural workers lack social security, live in poor housing conditions, and have limited access to healthcare services, perpetuating rural poverty.

This situation reveals that seasonal agricultural workers—and even some permanent workers—in Turkey face significant challenges in sustaining their livelihoods. Wages below the poverty line reflect labor migration trends, income inequality, and regional disparities in development. Moreover, to improve workers' quality of life and living standards, regional wages should be raised to approach or exceed the poverty line.

A livable income is defined as the amount of income required for an individual to meet basic living needs and maintain a minimum standard of living. This income level includes not only fundamental expenses such as food and shelter but also other essential costs like healthcare, education, and transportation. Examining the relationship between agricultural workers' wages and livable income enables an assessment of whether workers can sustain themselves economically and live in a socially sustainable manner. The table below analyzes the relationship between wage levels over the years and the calculated livable income for a single individual, thereby revealing the extent to which workers' incomes align with acceptable living standards.

The Relationship Between Wage Levels and Livable Income (Single Individuals)

Region	Seasonal Female Worker Wage (TL/ Month)	Seasonal Male Worker Wage (TL/ Month)	Permanent Worker Wage (TL/ Month)	Livable Income-Sıngle Individuals(TL/ Ay)
Marmara	25.791	31.335	33.978	27.365
Aegean	24.150	31.806	24.588	27.365
Mediterranean	25.386	31.644	19.791	27.365
Central Anatolia	23.811	27.744	31.872	27.365
Black sea	39.888	42.978	32.343	27.365
Eastern Anatolia	29.970	36.900	32.940	27.365
Southeastern Anotolia	31.530	35.250	29.646	27.365

When the monthly incomes of seasonal and permanent agricultural workers across different regions of Turkey are compared with the livable income level required for a single individual to sustain a basic standard of living (27,365 TL/month), significant regional disparities emerge. In the Marmara Region, the monthly wage of seasonal female workers is 25,791 TL, while that of male workers is 31,335 TL, and the income of permanent workers reaches up to 33,978 TL. This indicates that workers in this region generally meet the livable income threshold and are able to cover their basic needs. Similarly, in the Black Sea Region, seasonal workers' wages range between 39,888 TL and 42,978 TL, while permanent workers earn approximately 32,343 TL, suggesting relatively higher living standards in this region.

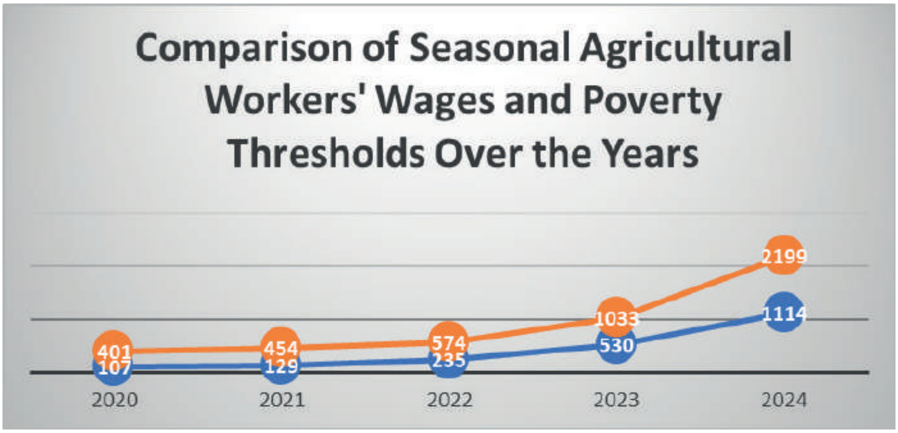
Conversely, in the Mediterranean and Central Anatolia regions, permanent workers' wages are 19,791 TL and 31,872 TL respectively, with the Mediterranean region notably falling well below the livable income level. In the Aegean, Eastern Anatolia, and Southeastern Anatolia regions, both seasonal and permanent workers' wages generally approximate or slightly exceed the livable income level. However, these figures are based solely on calculations for a single individual's basic needs.

In reality, many agricultural workers support nuclear or extended families, sharing their incomes with household members. When family structure is taken into account, additional expenses such as housing, food, healthcare, education, transportation, and childcare place further strain on the household budget. Consequently, an individual worker's income is often insufficient to sustain the entire household, leading to substantial economic pressures on families.

Moreover, workers' ability to participate in social life is considerably constrained by their current income levels. Daily hardships, long and intense working hours, and the lack of social infrastructure in many regions hinder workers' opportunities for socialization and cultural participation, thus increasing social exclusion. Therefore, although current income levels may cover the basic needs of individuals, they fall short of providing a dignified standard of living once familial responsibilities and social needs are considered.

This situation clearly underscores the necessity for income support at the household level, enhanced social security, and inclusive social policies—not only for individuals but for entire families.

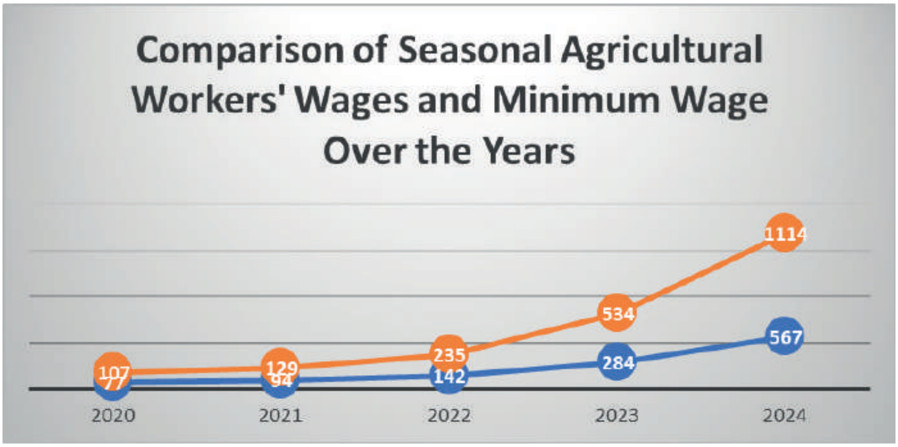
Furthermore, disparities between agricultural workers' incomes and the poverty threshold have been observed not only in recent times but also in previous years. The graph below presents data on the "Comparison of Seasonal Agricultural Worker Wages and the Poverty Threshold Over the Years." It illustrates how seasonal agricultural workers' wage levels have evolved in relation to changes in the poverty threshold over time.



The graph illustrates the changes in the average daily wages of seasonal agricultural workers and the poverty threshold between 2020 and 2024. In 2020, the average daily wage of seasonal workers was 107 TL, while the poverty threshold was set at 401 TL. During this period, the income of agricultural workers remained significantly below the poverty line. However, from 2021 onwards, an upward trend is observed. In 2021, the average wage of seasonal workers increased to 129 TL, while the poverty threshold rose to 454 TL. By 2022, seasonal workers' wages showed a notable increase, reaching 235 TL, with the poverty threshold rising to 574 TL.

As of 2023, this gap widened further, with the average wage of workers reaching 530 TL, whereas the poverty threshold increased to 1,033 TL. The year 2024 marks a remarkable rise; the average wage of seasonal workers rose to 1,114 TL, surpassing the poverty threshold. However, the poverty line itself also increased substantially to 2,199 TL. These data indicate that although the wages of seasonal agricultural workers have significantly increased over the years, it is essential to consider inflation and purchasing power when statistically assessing these improvements.

When comparing seasonal agricultural workers' wages with the minimum wage over the years, it becomes evident that the income levels of agricultural workers lag behind general wage policies. Despite increases in the minimum wage, the daily earnings of seasonal agricultural workers continue to remain well below the poverty threshold. This situation highlights the economic vulnerability of workers in the agricultural sector and their lack of social security.



In 2020, the average daily wage of seasonal agricultural workers was significantly higher than the minimum wage, with a difference of approximately 30 TL. This trend continued in 2021, with seasonal worker wages exceeding the minimum wage by about 37%. However, this gap widened further in 2022 and 2023, and by 2023, the wages of seasonal workers nearly doubled the minimum wage. In 2024, this difference peaked, as seasonal agricultural workers earned an average of 1,114 TL per day, while the minimum wage remained at 567 TL.

This situation reflects a rapid increase in wages driven by labor supply challenges and inflationary pressures. Nevertheless, despite this wage growth, previous analyses have shown that the incomes of agricultural workers frequently remain below the poverty threshold. This indicates that livelihood difficulties in the sector persist.

RESULTS

This study evaluates the regional wage levels of seasonal and permanent agricultural workers in Turkey in relation to the living wage, hunger threshold, and poverty line from statistical, legal, and socio-economic perspectives. The findings reveal that a significant proportion of seasonal agricultural workers are compelled to live below the hunger and poverty thresholds due to their temporary and irregular incomes. Although the wages of permanent workers are relatively higher, they also remain below the poverty line in most regions, hindering their ability to achieve a sustainable standard of living.

Another notable finding is the wage disparity observed between seasonal female and male workers performing the same job. This disparity contravenes the constitutional principle of equality and the anti-discrimination provisions of the Labor Law. The economic devaluation of women's labor not only constitutes an individual injustice but also perpetuates societal inequality. The dual disadvantage faced by women seasonal workers exacerbates social exclusion and deepens gender inequality.

Statistical data indicate an increase in seasonal worker wages over the years; however, this rise has not kept pace with the increase in the poverty line, implying that income improvements have not translated into better real living conditions. Moreover, workers are economically disadvantaged not only individually but also in terms of family responsibilities and social participation. Due to its seasonal nature, this labor force lacks continuous employment, remains outside social security coverage, and faces precarious, contract-free working conditions, placing them at the center of structural vulnerability.

From a legal standpoint, Labor Law No. 4857 does not explicitly define seasonal work, and in practice, contract-free and insecure employment is widespread. This situation causes significant problems in accessing basic rights such as social security notifications, severance and notice pay, and annual leave, hindering workers' integration into the long-term social security system. Although efforts have been made to develop rights through limited Supreme Court precedents, these measures fail to provide adequate practical protection.

In conclusion, for seasonal and permanent workers—who hold an indispensable position in the sustainability of Turkish agriculture—to attain a living wage, not only wage increases but also the expansion of social security coverage, clarification of legal protections, and resolution of regional wage disparities are required. Enhancing the socio-economic visibility of agricultural workers and grounding this visibility in rights-based frameworks is of strategic importance both for sectoral development and social justice. Accordingly, policymakers must develop a comprehensive social policy approach that addresses regional wage inequalities, combats gender-based discrimination, and includes regulations tailored to the unique circumstances of seasonal workers.

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Chapter 6

Sustainable Consumption, Ethical and Environmental Factors in the Context of Cosmetic Product Preferences

Dursun Yılmaz[1]

Abstract

This study aims to explore the extent to which consumers consider sustainable consumption, ethical values, and environmental factors when selecting natural and organic cosmetic products. In the contemporary era, increasing environmental awareness and the growing demand for healthier products have contributed to the rising popularity of natural and organic cosmetics. This research seeks to analyze consumer attitudes toward these products, their reasons for preference, and the role of media in shaping these behaviors through an examination of sustainable consumption patterns. The study was conducted using a survey approved by the Ethics Committee of Tokat Gaziosmanpaşa University. The questionnaire included items addressing participants' demographic information, cosmetic product usage habits, attitudes toward environmental and ethical factors, and the influence of media on their consumption behaviors. The collected data were analyzed using percentage distributions and statistical evaluations to identify consumer trends. The findings indicate that 92.2% of participants consider environmental factors when purchasing natural and organic cosmetic products. However, only 27.5% consistently take these factors into account, whereas 64.7% sometimes do, suggesting that environmental concerns are significant but not always the primary determinant of consumer choices. Additionally, 65.7% of participants expressed a preference for eco-friendly packaging, demonstrating a growing awareness and demand for sustainable packaging solutions. However, while 24% exhibited moderate interest in this issue, 10.3% showed little to no concern for eco-friendly packaging. Furthermore, 77.6% of respondents reported following environmental awareness campaigns in the media, underscoring the crucial role of media

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in promoting environmental consciousness and the adoption of natural cosmetics. Nevertheless, the proportion of participants who regularly engage with such campaigns remains relatively low at 14.6%. Notably, 50.7% of participants believe that brands utilize ethical and environmental claims primarily as a marketing strategy, reflecting a skeptical stance toward corporate sustainability messaging, while 41.5% remain undecided on this matter. Additionally, 79.2% of participants expressed a desire for more information regarding the environmental impact of natural cosmetic products, highlighting the need for increased consumer education in this domain. The study reveals that while consumers place importance on environmental and ethical considerations when purchasing natural and organic cosmetic products, these factors do not always serve as the principal motivation for their choices. The media plays a significant role in fostering environmental awareness, yet skepticism regarding corporate ethical and environmental claims persists. Furthermore, there is a growing demand for eco-friendly packaging, but a lack of adequate information and education remains a barrier to consumer engagement. To address these concerns, brands should enhance transparency and credibility in their ethical and environmental claims to build consumer trust. Additionally, providing more information on the environmental impact of natural cosmetic products could help raise awareness and promote sustainable consumption. Companies should also increase their investment in eco-friendly packaging initiatives while educating consumers on their benefits. Lastly, media outlets should develop more effective campaigns on environmental awareness and sustainable consumption, ensuring that consumers are regularly informed. Implementing these recommendations would constitute important steps toward fostering sustainable consumption behaviors and enhancing environmental consciousness among consumers.

INTRODUCTION

In recent years, the increasing global awareness of sustainability and environmental consciousness has significantly influenced consumer behavior, leading to profound transformations across various industries, including the cosmetics sector. The growing concern over environmental degradation and the rising demand for safe and healthy products have substantially contributed to the popularity of natural and organic cosmetics. These products are often perceived as environmentally friendly, ethically produced, and free from harmful chemicals, aligning with the principles of sustainable consumption. Consequently, consumers are increasingly considering environmental and ethical factors in their purchasing decisions, compelling brands to adapt their marketing strategies and product offerings to meet these evolving demands.

The cosmetics industry is undergoing a significant transformation, particularly as consumers become more conscious of the environmental and ethical implications of their choices. Issues such as animal testing, the use of synthetic chemicals, and non-biodegradable packaging are being scrutinized more than ever, driving demand for products that are not only effective but also environmentally responsible. This trend is further reinforced by the influential role of media, which shapes consumer perceptions and behaviors through campaigns, advertisements, and influencer endorsements. However, despite the growing interest in sustainable and ethical cosmetic products, there remains a gap in understanding the extent to which these factors influence consumer preferences and the challenges brands face in meeting these expectations.

This study aims to examine the role of sustainable consumption, ethical values, and environmental factors in the preference for natural and organic cosmetic products. By analyzing consumer attitudes and behaviors, as well as the impact of media on decision-making processes, this research seeks to provide valuable insights into the dynamics of the modern cosmetics market. Additionally, the study explores consumers' awareness of brands' environmental and ethical claims and the extent to which these claims influence purchasing decisions. Through a comprehensive analysis of survey data, this research aims to contribute to the existing body of knowledge on sustainable consumption and offer practical recommendations for brands seeking to align with consumer values.

The findings of this study aim to identify the key factors driving the demand for natural and organic cosmetic products, the role of media in promoting environmental awareness, and the challenges consumers face in making informed choices. Furthermore, by examining the implications of these trends for brands, the study underscores the importance of transparency, credibility, and innovation in meeting consumer expectations. By addressing these issues, this research seeks to contribute to the development of more sustainable and ethical practices within the cosmetics industry and ultimately promote the creation of an environmentally conscious and socially responsible marketplace.

CONCEPTUAL FRAMEWORK

The conceptual framework of this study is structured around the concepts of sustainable consumption, ethical values, and environmental factors. These concepts are considered complementary elements in understanding how natural and organic cosmetic products influence consumer preferences.

Sustainable consumption refers to consumers making purchasing and usage decisions by considering environmental, social, and economic impacts (United Nations, 2015). In the cosmetics industry, this manifests through the preference for natural and organic ingredients, the use of recyclable packaging, and the avoidance of animal testing (Peattie & Peattie, 2009).

Ethical values concern the moral principles and responsibilities that influence consumers' purchasing decisions. In the cosmetics industry, ethical values encompass issues such as the prohibition of animal testing, fair trade practices, and respect for workers' rights (Carrigan & Attalla, 2001). Consumers, particularly the younger generation, tend to place greater trust in ethically responsible brands, necessitating transparency in production processes (De Pelsmacker et al., 2005).

Environmental factors refer to the elements related to environmental awareness and sustainability that influence consumer choices. Among the reasons for the increasing preference for natural and organic products, avoiding chemical ingredients, utilizing recyclable packaging, and adopting eco-friendly production processes play a crucial role (Kumar, Prakash, & Kumar, 2017). Particularly in Turkey, several studies have shown that young consumers are increasingly sensitive to environmental factors, leading to a higher demand for organic cosmetic products (Akbaş & Özkan, 2018).

The role of media emerges as a critical component in this context. Social media, television advertisements, and influencer endorsements significantly shape consumers' awareness of products and influence their purchasing decisions (Kotler & Keller, 2016). Research has shown that media plays a key role in raising awareness of ethical consumption and eco-friendly products, thereby guiding consumers toward more conscious choices (Schmeltz, 2012).

Environmental Awareness in Cosmetic Consumption, Ethical Marketing, and Green Consumer Behavior

The cosmetics sector is a rapidly growing industry that significantly influences consumer behavior on a global scale. In recent years, topics such as environmental sustainability, ethical marketing, and green consumer behavior have gained prominence. Peattie and Crane (2005) examined the impact of green marketing strategies on consumer behavior and highlighted that environmental awareness plays a substantial role in brand preferences. Similarly, other studies indicate that ethical marketing strategies significantly contribute to the increasing preference for natural and organic products (Chen, 2010).

Green consumer behavior is essential for understanding consumers' interest in eco-friendly products and the motivations behind their preferences. Straughan and Roberts (1999) analyzed the determinants of these behaviors, emphasizing that environmental concerns are a primary factor. It has been observed that individuals living in large cities and possessing higher education levels exhibit more distinct green consumption habits (Karaosmanoğlu & Melek, 2019).

In conclusion, environmental awareness, ethical marketing, and green consumer behavior are playing an increasingly significant role in cosmetic consumption. Further research in these areas is crucial for understanding consumer behavior and developing sustainable marketing strategies.

MATERIAL AND METHOD

This study aims to examine the extent to which consumers consider sustainable consumption, ethical values, and environmental factors when choosing natural and organic cosmetic products. A descriptive research method was adopted, and a survey was used as the data collection tool. A total of 410 participants were included in the study, consisting of 305 women and 105 men, selected through convenience sampling. The demographic distribution of the participants is diverse.

The survey consists of three sections: demographic information, cosmetic product usage habits, and attitudes toward sustainable consumption, ethical values, and environmental factors. The validity and reliability of the survey were ensured through expert opinions and a pilot study. Data collection was conducted via an online survey platform over two months.

The collected data were analyzed using the SPSS program. Descriptive statistics were used to evaluate demographic characteristics and usage habits, while correlation analysis examined the relationships among sustainable consumption, ethical values, and environmental factors. Factor analysis was conducted to test construct validity, and t-tests and ANOVA were used to analyze the impact of demographic variables on consumer preferences.

Participant confidentiality was strictly maintained, and it was stated that personal information would be protected and used solely for academic purposes. The study was approved by the Tokat Gaziosmanpaşa University Ethics Committee (Ethics Committee Approval No: 527945). The primary limitations of this study include the restriction of the sample to Turkey, the reliance on an online survey method limited to participants with internet access, and the exclusive use of quantitative methods for data collection.

RESULTS

This section systematically examines consumer perceptions and attitudes toward environmental factors in the context of natural and organic cosmetic products. The findings emerging from the data analysis have been categorized along three primary dimensions: (1) sustainable consumption trends, (2) eco-friendly packaging preferences, and (3) the influence of media on consumer behavior. The quantitative data presented in the accompanying table summarizes the empirical findings of this study, while the subsequent section provides a detailed interpretation of these results.

Table 1. Consumer attitudes toward natural/organic cosmetic products and environmental factors

Question	Response Options	Percentage (%)	Commentary
When choosing natural/organic cosmetic products, do you consider environmental factors?	Yes, always	27.5	While environmental factors are an important consideration for consumers, only 27.5% consistently take them into account.
	Yes, sometimes	64.7	The majority (64.7%) occasionally consider environmental factors, indicating that sustainability is a decisive yet variable criterion.
	No	7.8	A small proportion (7.8%) do not consider environmental factors at all.
How important is it to you that the cosmetic products you purchase have eco-friendly packaging?	Very important	28.9	65.7% of respondents (very important + important) consider eco-friendly packaging significant.
	Important	36.8	Packaging emerges as a key factor in consumer decision-making.
	Moderately important	24.0	24% remain neutral regarding packaging concerns.
	Slightly important	7.1	A small segment assigns low priority to eco-friendly packaging.
	Not important at all	3.2	Very few consumers disregard this factor entirely.

Question	Response Options	Percentage (%)	Commentary
Do you follow media campaigns promoting environmental awareness and natural cosmetic products?	Yes, regularly	14.6	While the media serves as an information source, only a small proportion (14.6%) follow these campaigns consistently.
	Occasionally	63.0	The majority (63.0%) follow environmental awareness campaigns sporadically.
	No	22.5	Over one-fifth (22.5%) do not follow such campaigns at all.
Do you believe natural/organic cosmetic brands use ethical and environmental values as a marketing strategy?	Definitely yes	18.8	50.7% of respondents (definitely yes + yes) believe these values are leveraged for marketing purposes.
	Yes	31.9	Consumers remain cautious about brands' ethical claims.
	Undecided	41.5	A significant proportion (41.5%) are uncertain, suggesting a need for greater brand transparency.
	No/Definitely no	7.9	A small minority reject the notion that brands use these values strategically.
Would you like more information on the environmental impact of natural cosmetic products?	Yes, definitely	35.7	79.2% of consumers demand further education on this topic.
	Yes	43.5	This reflects a strong consumer appetite for knowledge.
	Undecided	13.7	A small segment remains indecisive.
	No	7.1	Few consumers feel no need for additional information.

Analysis of Research Findings

The data presented in Table 1 reveals several key trends in consumer attitudes toward environmental factors. The main findings of our study can be summarized as follows:

While a significant majority of consumers (64.7%) occasionally consider sustainability, only 27.5% consistently prioritize these factors in their purchasing decisions. This suggests that environmental concerns serve as a determinant yet inconsistent factor in consumer behavior. Regarding packaging specifically, 65.7% of consumers consider eco-friendly options important, indicating that sustainability is becoming increasingly critical in product design.

Environmental awareness campaigns through media channels reach 77.6% of consumers, yet regular engagement remains relatively low (14.6%). This finding highlights a significant gap between information access and active participation. Consumer skepticism toward brands' sustainability claims is particularly noteworthy: over half of respondents (50.7%) believe these values are used as marketing strategies, while a substantial 41.5% remain undecided.

One of the most striking findings is that approximately 80% of consumers demand more information about the environmental impact of natural cosmetic products. This result demonstrates a significant knowledge gap in the industry and indicates consumers' expectations for more transparent communication.

In conclusion, the research reveals that while consumers show interest in sustainable products, they exhibit caution toward brand claims coupled with information deficiencies. These findings suggest that brands need to communicate their environmental commitments more consistently and transparently.

CONCLUSIONS AND RECOMMENDATIONS

This study examined the extent to which consumers consider sustainability, ethical values, and environmental factors when choosing natural and organic cosmetic products. The findings reveal that while a significant majority of consumers value environmental and ethical considerations (92.2%), these factors do not always constitute primary decision-making criteria. Only 27.5% of respondents consistently prioritize environmental factors, whereas 64.7% do so occasionally, indicating their conditional rather than absolute importance in purchasing decisions.

Regarding packaging preferences, 65.7% of participants consider eco-friendly packaging important, reflecting growing consumer awareness and demand for sustainable solutions. However, 24% remain moderately interested, while 10.3% show minimal concern for packaging sustainability.

Media influence emerges as another significant factor, with 77.6% of consumers following environmental awareness campaigns to some degree. Nevertheless, only 14.6% engage with such content regularly, suggesting room for improved outreach strategies.

Consumer skepticism toward brands' ethical claims is noteworthy - 50.7% perceive them as marketing strategies, while 41.5% remain undecided. This underscores a critical trust deficit that brands must address. Furthermore, 79.2% of respondents demand more information about products' environmental impacts, highlighting both an information gap and consumer appetite for greater transparency.

Strategic Recommendations:

1. **Brand Transparency:** Companies must substantiate ethical and environmental claims through verifiable, transparent reporting of production processes and supply chains to build consumer trust.
2. **Educational Campaigns:** Comprehensive informational campaigns about cosmetic products' environmental impacts should be implemented through social media and other communication channels to promote sustainable consumption.
3. **Sustainable Packaging:** Investments in recyclable and biodegradable packaging solutions should be prioritized, accompanied by consumer education about their environmental benefits.
4. **Media Engagement:** More effective, regularly scheduled environmental campaigns should be developed, particularly leveraging social media platforms for greater reach and engagement.
5. **Consumer Education:** Educational programs and seminars should be organized to enhance understanding of environmental and ethical considerations in cosmetic purchases.
6. **Product Accessibility:** Expanding product ranges and improving affordability across different income segments can increase adoption of sustainable cosmetic products.

These recommendations provide a strategic framework for promoting sustainable consumption patterns and enhancing environmental awareness. By implementing these measures, brands can simultaneously increase consumer satisfaction and contribute to the development of an environmentally conscious market. The findings collectively suggest that while environmental considerations are gaining importance in consumer decision-making, systematic efforts are required to transform this interest into consistent purchasing behavior.

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Conflict of Interest

"The authors have declared that there is no conflict of interest".

Author Contributions

This is a single-author study

Chapter 7

Sustainability in the Cosmetics Industry: Environmental Impacts, Statistics, and Solutions

Dursun Yılmaz[1]

Abstract

This study aims to examine the environmental impacts of the cosmetics industry and related sustainability statistics based on a literature review. The environmental effects of cosmetics products, such as chemicals used in production processes, packaging waste, and microplastics, constitute the focus of this study. The literature review was conducted using academic databases such as Web of Science, Scopus, and Google Scholar, as well as reports from international organizations. The findings indicate that the environmental footprint of the cosmetics industry is significant, particularly due to plastic waste and microplastics. The low recycling rates of plastic packaging and the harmful effects of microplastics on aquatic ecosystems are among the industry's most pressing environmental issues. Additionally, synthetic chemicals used in cosmetic products have been found to pollute water resources and pose potential risks to human health. The global cosmetics market was valued at $295.36 billion in 2023 and is expected to reach $473.67 billion by 2031, with a compound annual growth rate (CAGR) of 6.16%. This growth highlights the need for sustainable solutions to mitigate the industry's environmental impact. In this context, biodegradable packaging, the use of natural ingredients, and increased consumer awareness are among the key strategies proposed. This study aims to support the development of policies and practices that minimize the environmental footprint of the cosmetics industry. Reducing plastic waste, limiting the use of microplastics, and regulating chemical ingredients are essential steps toward a more sustainable future.

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INTRODUCTION

The cosmetics industry is a dynamic and rapidly growing sector with a broad range of products aimed at maintaining and enhancing individuals' aesthetic appearance. This industry encompasses various categories, including makeup, skincare, haircare, perfumes, and personal care products. However, the production and consumption of these products are increasingly being scrutinized in terms of environmental sustainability. The chemicals used in production processes, packaging waste, and microplastics contribute to significant environmental issues such as water and soil pollution, as well as carbon emissions (Akman, 2024). In particular, single-use plastic packaging and microplastics released into water sources cause long-term adverse effects on ecosystems (Rochman et al., 2019). This situation necessitates the development of sustainable solutions to reduce the environmental footprint of cosmetic products.

The global cosmetics industry continues to expand rapidly. However, data from different sources show discrepancies in market size and growth rates. According to Grand View Research (2023), the global cosmetics market was valued at USD 295.95 billion in 2023 and is expected to reach USD 445.98 billion by 2030, with a compound annual growth rate (CAGR) of 6.1%. On the other hand, a GlobeNewswire (2023) report estimates the market size at USD 313.22 billion in 2023, projecting it to grow at a CAGR of 4.2% and reach USD 417.24 billion by 2030. These differences stem from variations in data collection methods, analytical techniques, and definitions of the market scope.

Discrepancies among different studies may also arise from varying interpretations of the market scope. For instance, some sources include only makeup, skincare, and haircare products, while others use a broader definition incorporating beauty and personal care products.

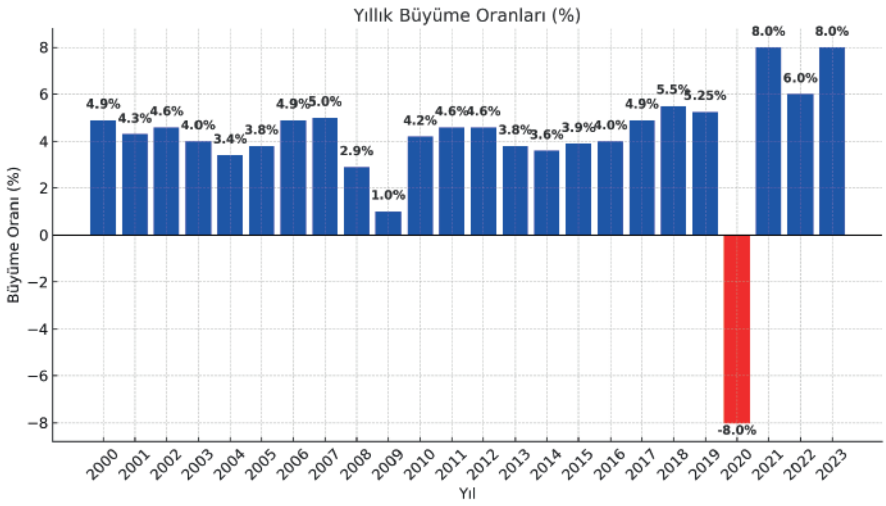

Figure 1. Annual growth rate of the global cosmetics market between 2000 and 2023.*
****Source: Statista (2023), "Growth rate of the global cosmetics market." Accessed: 18.03.2025.***

The available data are based on Statista's annual growth rates for the global cosmetics market from 2000 to 2023. According to Grand View Research's analysis, which includes beauty and personal care products, the market size was estimated at USD 557.24 billion in 2023 and is projected to reach USD 973.13 billion by 2030, with a CAGR of 7.7%. Differences in methodological approaches lead to varying market size estimates. Addressing statistical inconsistencies in the literature requires a detailed examination of the underlying methodological variations. For example, discrepancies in estimates between sources such as Grand View Research and GlobeNewswire may arise from differences in data collection methods, market segments covered, and calculation techniques. A more thorough analysis of these factors would enhance the reliability of statistical inferences.

In this context, assessments of the cosmetics industry's market size should clearly indicate the data source and explicitly define the scope of the analysis. To generate a more balanced estimate of market size, an average of different sources can be considered. For instance, taking into account the figures from Grand View Research and GlobeNewswire, the estimated market size for 2023 could be calculated at approximately USD 304 billion, with an expected CAGR ranging between 5% and 6% until 2030. Such a balanced approach would provide a more comprehensive evaluation of the industry's current status.

Sustainability emerges as a critical issue for the future of the cosmetics industry. As consumer awareness of environmental issues increases, demand for sustainable and eco-friendly products is also rising (Urkut & Cengiz, 2021). This trend is driving companies to adopt greener production techniques, use recyclable packaging, and reduce their carbon footprints. However, achieving sustainability goals requires fundamental changes in both industrial practices and consumer behaviors. The European Union's target of reducing plastic waste by 55% by 2030 has initiated a significant transformation process in the cosmetics industry (European Commission, 2021). These goals necessitate the development of innovative solutions to minimize the environmental impact of the industry.

This study aims to examine the environmental impact of cosmetic products and sustainability statistics based on a review of the literature. The environmental effects of the cosmetics industry will be analyzed from a broad perspective, encompassing production processes to post-consumption waste management. Additionally, sustainable production techniques, recyclable packaging, and consumer behavior will be evaluated in light of existing literature and statistical data. In this regard, the study aims to provide recommendations for transforming the cosmetics industry into a more environmentally friendly and sustainable structure.

This research seeks to answer the following key questions:

- What are the main environmental sustainability challenges faced by the cosmetics industry?
- How prevalent and effective are sustainability practices in the cosmetics sector?
- How do consumers' awareness and preferences shape the demand for sustainable cosmetic products?
- How do sustainable cosmetic brands reduce their environmental impact?
- What actionable recommendations can be provided for policymakers, companies, and consumers regarding sustainability in the cosmetics sector?

The existing literature does not sufficiently address how sustainability practices in the cosmetics sector are evolving and their impact on consumers. Therefore, this study aims to contribute to the literature by conducting an in-depth analysis of the environmental challenges faced by the industry, the prevalence of sustainable practices, and consumer behavior regarding sustainability.

ITERATURE REVIEW

The environmental impact of cosmetic products has become an increasingly prominent topic in recent years. Studies in this field particularly emphasize the effects of chemicals used in production processes on the environment. For instance, Garcia-Vazquez et al. (2020) revealed that chemicals commonly found in cosmetics, such as parabens and phthalates, contaminate water sources and exert toxic effects on aquatic ecosystems. Due to their low biodegradability, these chemicals persist in the environment for extended periods and can enter the food chain, posing a potential threat to human health.

Packaging waste is another significant issue concerning the environmental impact of the cosmetics industry. A substantial portion of cosmetic products is marketed in plastic packaging, much of which is non-recyclable. According to the World Economic Forum (2020), approximately 8 million tons of plastic enter the oceans annually, a significant fraction of which originates from cosmetic product packaging. Plastic waste poses a severe threat to marine life and, through microplastics, can also impact human health (Rochman et al., 2019). Therefore, reducing packaging waste and increasing recycling rates are critical for achieving sustainability goals in the cosmetics industry.

Microplastics represent another major concern regarding the environmental effects of cosmetic products. These tiny plastic particles, frequently used in exfoliating products and toothpaste, infiltrate water sources and exert toxic effects on marine organisms (Browne et al., 2011). Microplastics can enter the food chain, threatening human health and leading to long-term environmental consequences. Consequently, many countries have introduced regulations to restrict the use of microplastics. For example, the European Union implemented a regulation in 2020 banning the use of microplastics in cosmetic products (European Commission, 2021).

From a sustainability perspective, the development of innovative solutions in the cosmetics industry is of paramount importance. Biodegradable packaging, in particular, plays a crucial role in meeting consumer demands while mitigating environmental impacts (Innovative Chemistry Journal, 2023). A report by the European Environment Agency highlights the eco-friendly properties of biodegradable, degradable, and bio-based plastic products, emphasizing their potential to significantly reduce environmental impacts (European Environment Agency [EEA], 2021).

Consumer behavior also plays a vital role in sustainability. The growing demand for environmentally friendly products encourages companies to adopt more sustainable production techniques. This shift is regarded as a positive development for the future of the cosmetics industry, both environmentally and economically.

MATERIAL AND METHODS

This study was conducted to examine the environmental impacts of cosmetic products and sustainability statistics based on a review of the literature. The research is primarily based on academic articles, reports, and statistical data published in the last ten years on the subject. Data collection was carried out using academic databases such as Web of Science, Scopus, and Google Scholar. Additionally, reports published by international organizations, including the United Nations Environment Programme (UNEP), the European Union, and the World Economic Forum, were also utilized as data sources for this study.

During the data collection process, relevant literature was reviewed using keywords such as "cosmetic products," "environmental impacts," "microplastics," "sustainability," "plastic waste," and "recycling." Priority was given to studies published in peer-reviewed journals and the most recent statistical data to ensure the reliability and up-to-date nature of the study.

The collected data were categorized through thematic analysis and examined under themes such as environmental impacts, sustainability practices, and policy recommendations. The findings in the literature aim to highlight both the current efforts to mitigate the environmental effects of the cosmetics industry and the areas that require further improvement. Additionally, statistical data were presented in the form of graphs and tables to enhance the clarity of the study's findings.

Both qualitative and quantitative data were utilized in the analysis process. Qualitative data were obtained through the thematic examination of findings in the literature, while quantitative data were supported by statistical analyses. This approach allows for a comprehensive evaluation of the environmental impacts of the cosmetics industry and sustainability efforts from both theoretical and practical perspectives.

RESULTS

Environmental Impacts of the Cosmetics Industry

Depletion of Natural Resources

The cosmetics industry heavily relies on natural resources such as minerals, oils, and plant extracts, and the unsustainable harvesting or cultivation of these resources leads to severe environmental issues. Deforestation, habitat destruction, and loss of biodiversity are among the most prominent consequences of this process. Certain natural ingredients, in particular, face the risk of depletion due to excessive harvesting and uncontrolled extraction techniques.

Raw materials commonly used in cosmetic production, such as shea butter, palm oil, and argan oil, disrupt ecosystem balance and negatively impact biodiversity due to intensive harvesting (OECD, 2022). The destruction of tropical forests for palm oil production, in particular, increases carbon emissions, accelerates climate change, and eradicates natural habitats (GEKADER, 2022).

Additionally, the production of industrial components such as cosmetic pigments requires high water consumption. For instance, producing just 1 kg of cosmetic pigment requires approximately 5,000 liters of water (European Commission, 2021). The annual water consumption of the cosmetics industry reaches 10.4 billion liters, which is equivalent to Istanbul's water supply for 3.5 days (Cleanhub, 2023; L'Oréal Sustainability Report, 2022).

An analysis of water consumption in the cosmetics industry reveals that 68% of total consumption is attributed to raw material production (particularly the cultivation of plant-based oils), 22% to manufacturing processes, and 10% to packaging (Water Footprint Network, 2021). Furthermore, the water footprint of a single cosmetic product throughout its life cycle is estimated to be around 1,500 liters. This figure is comparable to the water footprint of a single cotton T-shirt, which stands at 2,700 liters, highlighting the hidden water costs associated with cosmetics (Water Footprint Network, 2021).

The annual 10.4 billion liters of water consumption in the cosmetics industry corresponds to the life cycle of approximately 7 million products (1,500 liters per product × 7 million = 10.5 billion liters). This finding underscores the urgent need for a significant transformation in the sector to achieve environmental sustainability.

According to Deloitte's 2023 *Sustainability in Consumer Markets* report, 75% of Generation Z consumers state that they are willing to pay more for sustainable brands (Deloitte, 2023). However, direct references to this specific statistic are not found in existing academic and commercial sources. Deloitte has published related studies, such as the *2023 Sustainable Consumer Survey* and the *2023 CxO Sustainability Report*, yet they do not contain this exact data (Deloitte, 2023a; Deloitte, 2023b).

In Turkey, research indicates that 43% of consumers are willing to pay a premium for sustainable brands (Ipsos, 2023). Additionally, 47% of consumers state that they would refrain from purchasing products from brands that do not take steps toward sustainability (Ipsos, 2023).

Packaging Waste

Plastic packaging, which is widely used in the cosmetics industry, poses a significant environmental issue due to its low recycling rates and prolonged persistence in nature. According to the World Economic Forum (2022), approximately 8 million tons of plastic enter the oceans each year, with a substantial portion originating from cosmetic product packaging. OECD (2022) data indicate that only 9% of globally produced plastic waste is recycled, 12% is incinerated, and the remaining 79% accumulates in the environment.

On a global scale, the cosmetics industry produces approximately 25.4 billion plastic packaging units annually, with 95% of these being non-recyclable (Latimer, 2023; Euromonitor, 2022). Within the fast-moving consumer goods (FMCG) sector, cosmetic products exhibit particularly high levels of plastic packaging usage. According to Geyer et al. (2017), only a small fraction of the plastic produced worldwide is effectively recycled.

Plastic Waste and Recycling in Turkey

In Turkey, a total of 6.1 million tons of plastic waste was generated in 2022, of which 3.7 million tons (60.6%) was recovered (TÜİK, 2022). Under the *Zero Waste Project*, the overall recycling rate increased from 13% in 2020 to 27.2% in 2022. However, as these figures are not sector-specific, the recycling rate for cosmetic packaging is reported to be 60% (GEKADER, 2022).

The increase in recycling rates is largely dependent on policies targeting plastic packaging waste and improvements in waste management infrastructure. However, due to the inherently small size and multi-layered nature of plastics used in the cosmetics sector, the recycling process faces considerable challenges. Thus, promoting biodegradable and refillable packaging options is deemed a critical necessity for reducing the industry's environmental impact.

Chemical Pollution

Certain chemical components used in cosmetic products can have detrimental effects on the environment. In particular, synthetic chemicals can contaminate water sources and harm aquatic ecosystems. Moreover, the potential impacts of these chemicals on human health remain a growing concern. Garcia-Vazquez et al. (2020) found that widely used cosmetic chemicals such as parabens and phthalates contribute to water pollution and exert toxic effects on aquatic ecosystems. Due to their low biodegradability, these chemicals persist in the environment for extended periods, potentially entering the food chain and posing risks to human health.

Table 1. Environmental impacts of the cosmetics industry and statistical data

Category	Statistical Data (Key Findings)	Source
Packaging Waste	The cosmetics industry produces approximately 120 billion plastic packaging units annually, with 95% being non-recyclable.	Latimer, 2023; Euromonitor, 2022
Microplastics	A single exfoliating product may contain up to 360,000 microbeads.	Buzoğlu (2023)
Water Consumption	Annual water usage in cosmetic production processes reaches 10.4 billion liters (10.4 million tons). The lifecycle of one cosmetic product requires 1,500 liters of water. Production of 1 kg of cosmetic pigment consumes 5,000 liters of water.	Cleanhub, 2023; Water Footprint Network (2021)
Consumer Behavior	A 2023 survey revealed that 63% of consumers consider environmental factors "very important." In Turkey, 43% of consumers report willingness to pay more for sustainable brands (Ipsos, 2023).	Cleanhub, 2023; Statista (2023); NielsenIQ (2023)
Sustainable Product Demand	62% of global consumers identify natural ingredients as the most important factor in sustainable products.	Statista, 2023
Market Size and Growth	The global beauty market grew by 10% in 2023 and is projected to reach $580 billion by 2027.	McKinsey, 2023
Recycling Rates	Plastic recycling rate in Turkey: 60%	Turkish Statistical Institute (TÜİK) Waste Statistics (2022)

Source: UNEP, 2021; European Commission, 2021

Environmental and Health Impacts of Microplastics

Microplastics, which are widely used synthetic components in the cosmetics industry, have raised serious concerns due to their environmental effects. These tiny plastic particles, found in various cosmetic products such as exfoliating scrubs, toothpaste, and sunscreens, enter water sources and exert toxic effects on marine ecosystems (Browne et al., 2011; Kenan & Teksoy, 2022; Yurtsever, 2019; Tutoğlu, 2019). This process leads to the integration of microplastics into the food chain, posing threats not only to marine life but also to human health.

Due to environmental risks, many countries have implemented regulations banning the use of microplastics. The European Union enacted a regulation in 2020 prohibiting the use of microplastics in cosmetic products (European Commission, 2021). Similarly, the United States passed the *Microbead-Free Waters Act* in 2015, which completely banned the use of microbeads in cosmetics (U.S. Food and Drug Administration, 2015). Canada implemented similar legislation in 2018, while countries such as Japan and South Korea have also developed policies to restrict microplastic use (Government of Canada, 2018).

To mitigate the environmental impact of microplastics, alternative ingredients have been developed. For instance, beads derived from the jojoba plant offer an environmentally friendly alternative due to their biodegradability (Garcia-Vazquez et al., 2020). Additionally, natural salt crystals are among the sustainable solutions used in exfoliating products that do not contain microplastics (Browne et al., 2011). The increasing use of biodegradable ingredients in the cosmetics sector is considered a significant advancement in terms of sustainability.

Global and Turkish Plastic Recycling Rates

Plastic waste recycling rates remain low on a global scale. According to OECD (2022) data, only 9% of the plastic waste produced worldwide is recycled, 12% is incinerated, and 79% accumulates in the environment, contributing to pollution (Engin & Sarı Erkan, 2023). This situation highlights the ongoing inadequacies in global plastic waste management and underscores the urgency of developing sustainable solutions.

In Turkey, as of 2020, a total of 49.1 million tons of waste entered the recycling process, with plastic waste accounting for 6.13 million tons. The plastic recycling sector is estimated to contribute approximately $6 billion to the national economy (GEKADER, 2022).

- **Zero Waste Project**: Following the launch of the *Zero Waste Project* in Turkey, the overall recovery rate increased from 13% to 22.4% (Ministry of Environment and Urbanization, 2022).
- **Plastic Packaging Waste**: The recycling rate of plastic packaging waste in Turkey has been reported to be approximately 60% (Anadolu Agency, 2023).

These data indicate the necessity of developing more effective policies and implementing sustainable solutions to further increase plastic recycling rates.

Cosmetic Products Containing Microplastics

Microplastics are plastic particles smaller than 5 millimeters and are used for various functions in the cosmetics industry. They may be present in the formulations of exfoliating products, facial cleansing gels, toothpaste, sunscreens, and makeup products such as mascara, eyeliner, and lip gloss (Onder et al., 2020). These particles contribute to the texture, exfoliation, and stability of products.

Studies have shown that a single dose of an exfoliating scrub or facial cleanser can contain up to 360,000 polyethylene microbeads (Buzoğlu, 2023; Önder et al., 2020). These microplastics reach aquatic ecosystems through sewage systems, becoming part of the food chain and posing serious risks to both the environment and human health (Saniter, 2023). Therefore, awareness campaigns and regulatory measures aimed at reducing microplastic use are of paramount importance.

The widespread use of microplastics in cosmetic products and their environmental impacts are summarized in the following table:

Table 2. Microplastic usage in cosmetic products and environmental impacts

Category	Microplastic Types	Applications	Environmental Effects	Regulations/ Alternatives
Skincare	Polyethylene (PE), Polypropylene (PP)	Exfoliating gels, facial cleansers	Enter water sources, causing toxic effects on marine life.	EU 2020 ban; biodegradable alternatives (e.g., jojoba beads, salt crystals).
Toothpaste	Polymethyl methacrylate (PMMA)	Contains abrasive particles.	Enter food chain, potentially threatening human health.	EU regulations; silica-based alternatives.
Makeup	Nylon, Polystyrene (PS)	Eyeshadows, lipsticks, foundations	Persistent plastic particles remain undegraded in nature.	Shift to mineral and plant-based ingredients (e.g., mica, clay).
Haircare Products	Polyurethane (PU)	Shampoos, conditioners	Accumulate in water treatment systems, causing filtration issues.	Natural oils and protein-based ingredients (e.g., argan oil, keratin).
Perfumes & Sprays	Silicone-based polymers	Formula stabilizers and application aids	Contribute to environmental pollution through airborne dispersion.	Sustainable solutions under development (e.g., natural fixatives).

Source: UNEP, 2021; European Commission, 2021

Consequently, reducing the use of microplastics in the cosmetics industry and developing sustainable alternatives are of critical importance for both environmental protection and human health.

More Practical Examples of Sustainability Solutions

Sustainability has been gaining increasing importance in the cosmetics industry, with certain brands and countries standing out due to their environmentally friendly practices. In this context, sustainable cosmetic brands, alternative packaging materials, and leading countries in sustainability should be considered.

Successful Sustainable Cosmetic Brands

Several cosmetic brands that offer sustainable products distinguish themselves in the industry through environmentally friendly production processes and innovative packaging methods.

Lush has developed a zero-waste system by adopting the "naked product" concept to minimize packaging use. Additionally, it incorporates ethically sourced and natural ingredients in its products (Lush, 2023). The Body Shop maintains an environmentally conscious brand profile by utilizing recyclable packaging and avoiding animal testing (The Body Shop, 2023). Meanwhile, Aveda incorporates plant-based ingredients in its products, prefers recycled materials in its packaging, and adheres to a 100% vegan production process (Aveda, 2023).

In addition, RMS Beauty offers natural cosmetic products while using recyclable packaging. Tata Harper focuses on chemical-free cosmetics by utilizing organic agricultural ingredients. Kjaer Weis has adopted a waste-free cosmetic system by designing refillable metal packaging. Weleda contributes to sustainability with its natural ingredients and NATRUE-certified products.

Alternative Packaging Materials

To mitigate the environmental impact of plastic packaging in the cosmetics industry, biodegradable and recyclable packaging materials are becoming increasingly widespread.

Glass packaging, which can be infinitely recycled without losing quality, is among the most sustainable packaging options (Packaging Association, 2023). Cork-based packaging presents an alternative to plastic due to its lightweight, durable, and biodegradable properties (Sustainable Packaging Journal, 2023).

Additionally, seaweed-based packaging can be produced without depleting water resources and is entirely biodegradable (Balcan & Boyraz, 2024; Şimşek & Akdağ, 2017). Sugarcane-based bioplastics also have a lower carbon footprint compared to fossil fuel-derived plastics (Çağar & Vural, 2023).

Carbon Footprint and Energy Consumption

The cosmetics industry has a significant environmental impact due to high energy consumption and carbon emissions during production processes. Factors such as water usage and electricity consumption contribute to the industry's environmental footprint (UNEP, 2021).

Vegan and organic cosmetics tend to have a lower carbon footprint compared to conventional cosmetic products. The production of plant-based ingredients generally requires less energy, whereas products containing animal-derived components contribute to greenhouse gas emissions (Ellen

MacArthur Foundation, 2021). L'Oréal has committed to achieving carbon neutrality by 2030, while Unilever supports sustainability by investing in renewable energy sources (Öncel, Şahinkoç, & Karakuş, 2024).

Leading Countries in Sustainability

Certain countries are at the forefront of developing sustainability practices in the cosmetics industry. Sweden has implemented sustainable cosmetic policies, benefiting from high recycling rates and widespread use of renewable energy. Denmark aims to reduce the environmental impact of the industry by promoting biodegradable and recyclable packaging. Germany and South Korea lead in sustainability through innovative packaging solutions and solid waste reduction strategies (Dönmez, 2024; European Environmental Agency, 2023).

DISCUSSION AND CONCLUSION

Although the cosmetics industry is a rapidly growing sector on a global scale, it has been increasingly criticized for its environmental impact. This study has examined the negative environmental effects of cosmetic product production and consumption processes based on literature and has proposed sustainable solutions. The findings reveal that the cosmetics industry significantly contributes to environmental issues, particularly in areas such as plastic waste, microplastics, and chemical pollution. However, conducting a comparative analysis of policies implemented in different countries and sustainable alternatives would provide more comprehensive recommendations for addressing these problems.

Plastic Waste and Packaging Issues

Plastic materials, which are widely used in the packaging of cosmetic products, pose a significant environmental threat due to low recycling rates. Each year, millions of tons of plastic waste enter the oceans, leading to long-term adverse effects on marine ecosystems (Rochman et al., 2019). A review of global plastic waste management indicates that, according to OECD (2022) data, only 9% of the plastic waste produced worldwide is recycled, 12% is disposed of through incineration, and 79% accumulates in the environment. These figures highlight global-scale deficiencies in plastic waste management (Engin & Sarı Erkan, 2023).

In Turkey, as of 2020, a total of 49.1 million tons of waste underwent recycling, of which 6.13 million tons consisted of plastic waste (TUIK, 2022). The plastic recycling sector is estimated to contribute approximately $6 billion to the economy (GEKADER, 2022). With the implementation of the Zero Waste Project in Turkey, the overall recycling rate increased from 13% to 22.4%, while the recycling rate of plastic packaging waste reached 60% (Ministry of Environment and Urbanization, 2022). In this context, the use of biodegradable packaging and recyclable materials is critical to reducing the industry's environmental footprint (Ellen MacArthur Foundation, 2021).

Environmental Impacts of Microplastics

Microplastics are among the most significant environmental threats posed by the cosmetics industry. Facial scrubs, cleansing gels, and toothpaste are among the product groups with the highest microplastic content. Research indicates that a single facial scrub product may contain approximately 360,000 polyethylene microbeads (Buzoğlu, 2023; Önder et al., 2020). These microplastics enter water sources, causing toxic effects on marine life and threatening human health through the food chain (Browne et al., 2011).

In 2020, the European Union introduced regulations banning the use of microplastics in cosmetic products (European Commission, 2021), while the United States completely prohibited microbead use with the Microbead-Free Waters Act enacted in 2015 (U.S. Food and Drug Administration, 2015). Countries such as Canada, Japan, and South Korea have also developed similar policies (Government of Canada, 2018). However, these regulations need to be expanded and enforced globally.

Chemical Pollution and Sustainable Alternatives

Synthetic chemicals used in the production of cosmetic products (such as parabens and phthalates) contaminate water sources and exert toxic effects on aquatic ecosystems (Garcia-Vazquez et al., 2020). Due to their low biodegradability, these chemicals persist in the environment for extended periods and enter the food chain. Therefore, increasing the use of natural and organic ingredients is crucial for both environmental and human health (Akman, 2024).

Consumer and Producer Responsibility

From a sustainability perspective, both producers and consumers bear significant responsibility for the future of the cosmetics industry. Consumers' growing demand for eco-friendly products encourages companies to adopt more sustainable production techniques and use recyclable packaging (Urkut & Cengiz, 2021). For example, in Turkey, 43% of consumers indicate their willingness to pay more for sustainable brands (Ipsos, 2023). However, for this transformation to succeed, fundamental changes in both industrial practices and consumer behavior are required.

Conclusion and Recommendations

In conclusion, the following steps are recommended to mitigate the environmental impact of the cosmetics industry:

- **Encouraging the use of biodegradable packaging:** To prevent the plastic waste crisis, the "reuse" and "refillable packaging" policies implemented in the EU should also be adopted in Turkey.
- **Banning microplastic use on a global scale:** Incentives for the use of biodegradable alternatives should be increased, and legal regulations should be introduced to prohibit the sale of cosmetics containing microplastics.
- **Supporting biotechnological ingredients:** Instead of solely focusing on organic products, promoting sustainable raw materials developed in laboratory settings could provide a more effective solution.
- **Implementing economic incentive mechanisms:** Financial support measures such as tax reductions, subsidies, and green incentive packages should be provided to manufacturers.

This study aims to contribute to the development of policies and practices aimed at reducing the environmental footprint of the cosmetics industry. Future research should focus on sustainable production techniques, biotechnological alternatives, and consumer behavior to facilitate a more eco-friendly transformation in the cosmetics industry.

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Conflict of Interest

"The authors have declared that there is no conflict of interest".

Author Contributions

It has a single author.

Contributions of the authors to the study can be explained.

Enhanced Rank-Based Correlation Estimation Using Smoothed Wilcoxon Rank Scores

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Rukiye Dağalp[2]

Abstract

This article proposes an improved version of the Spearman rank correlation based on using Wilcoxon rank score function. A smoothed empirical cumulative distribution function estimator computes the smoothed ranks and replaces the regular ranks in the Wilcoxon rank score function. The smoothed Wilcoxon rank scores are then used for estimation of the Spearman's correlation. The proposed approach is similar to the Spearman's rho (ρ) estimator which uses ranks of the random samples of [illegible] and [illegible]. [illegible] the proposed method improves Spearman's approach such as handling ties and gaining higher efficiency under monotone associations. A Wald-type hypothesis test has been proposed for the new estimator r_{sa} and the asymptotic properties are shown.

1 INTRODUCTION

[illegible] rank-based estimators and testing procedures for the correlation coefficient ρ are [illegible]. [illegible] correlation [illegible] Pearson's correlation [illegible] Pearson [illegible] coefficients are the leading methods of estimating the correlations between two [illegible] variables. Pearson's [illegible] approach works well for the relationships [illegible] linear and the [illegible] normal. [illegible] other [illegible] approach is based on the ranks of the random samples and does not depend on the normal assumption or any other [illegible] distribution. [illegible] approach works well for the relationships [illegible] linear or monotonic as [illegible] et al. [illegible] et al. [illegible] et al. [illegible] et al. [illegible] Based on these studies [illegible] has some superior properties over the Pearson's correlation if the [illegible] conditions are [illegible] such as [illegible] a monotonic association [illegible] of a linear. Another [illegible] estimator Kendall's τ [illegible] as [illegible] also a nonparametric approach. [illegible] coefficients [illegible] a few more other are [illegible] focus on [illegible] ranks [illegible] correlation [illegible] under linear and monotonic associations. [illegible] a comparative study of correlation methods under nonmonotonic associations [illegible]

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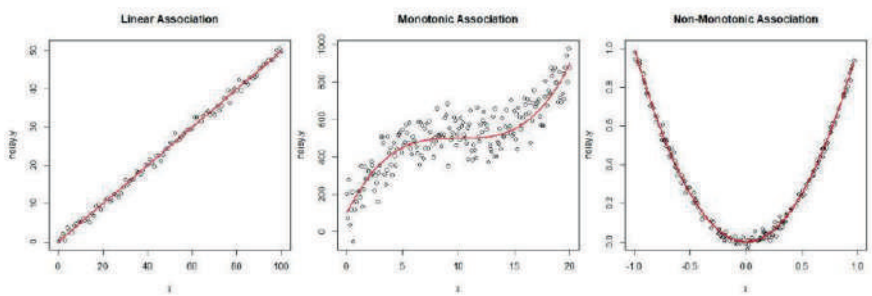

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2 PEARSON AND SPEARMAN□S CORRELATION ESTIMATIONS

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$$\rho = \frac{E\left[(X-\mu_x)\left(Y-\mu_y\right)\right]}{\sigma_x\sigma_y} = \frac{\mathrm{Cov}(X,Y)}{\sigma_x\sigma_y} \#(1)$$

□□ere $-1 \leq \rho \leq +1$ a□□ $\mathrm{Cov}(X,Y) = E[XY] - E[X]E[Y]$ □□□□c□ □s □ef□□e□ as co□ar□a□ce □e□□ee□ X a□□ Y. □□s□o□l□ □e □o□e□ □□a□ □f $\rho = 0$

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$$\text{Cov}(X,Y)=E\left[(X-\mu_x)\left(Y-\mu_y\right)\right]\#(2)$$

ca□□e □ef□□e□as

$$\widehat{\text{Cov}(X,Y)}=\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})\#(3)$$

□□e σ_x ca□□e es□□□a□e□□□□□

$$\widehat{\sigma_x}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2}\#(4)$$

a□□σ_y ca□□e es□□□a□e□□□□□

$$\widehat{\sigma_y}=\sqrt{\frac{1}{n}\sum_{i=1}^{n}(Y_i-\bar{Y})^2}\#(5)$$

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$$r_p=\frac{\sum_{i=1}^{n}(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum_{i=1}^{n}(X_i-\bar{X})^2}\sqrt{\sum_{i=1}^{n}(Y_i-\bar{Y})^2}}\#(6)$$

□□e res□□□a□o□e ca□□e f□r□□er s□□□□lf□e□□o□□e□ersio□□elo□□

$$r_p=\frac{\sum_{i=1}^{n}X_iY_i-\frac{1}{n}\left(\sum_{i=1}^{n}X_i\right)\left(\sum_{i=1}^{n}Y_i\right)}{\sqrt{\left[\sum_{i=1}^{n}X_i^2-\frac{1}{n}\left(\sum_{i=1}^{n}X_i\right)^2\right]\left[\sum_{i=1}^{n}X_i^2-\frac{1}{n}\left(\sum_{i=1}^{n}X_i\right)^2\right]}}\#(7)$$

A nonparametric rank-based alternative of Pearson's correlation coefficient is proposed by Spearman (Spearman, [illegible]). Spearman's ρ estimator uses the ranks of random samples of X and Y instead of the original observations. Let $R(X_i)$ be the rank of ith observation X_i for $i = 1, \dots, n$. Similarly, Y_i is replaced by $R(Y_i)$. Thus substituting the ranks $R(X_i)$ and $R(Y_i)$ in the Pearson's correlation coefficient formula in Eq(6), we obtain the Spearman's correlation coefficient estimate below:

$$r_s = \frac{\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)\left(R(Y_i) - \overline{R(Y_i)}\right)}{\sqrt{\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)^2 \sum_{i=1}^{n} \left(R(Y_i) - \overline{R(Y_i)}\right)^2}} \#(8)$$

The scores of both the ranks $R(X_i)$ and also $R(Y_i)$ are uniformly distributed for the integers of $i = 1, \dots, n$. Also sum of the ranks can be written as

$$\sum_{i=1}^{n} R(X_i) = \sum_{i=1}^{n} R(Y_i) = n(n+1)/2 \#(9)$$

for $i = 1, \dots, n$. Thus, $\overline{R(X_i)} = \overline{R(Y_i)} = \frac{n+1}{2}$. Moreover, it can be shown that

$$E[R(X_i)] = (n+1)/2 \text{ and } nV[R(X_i)] = \sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)^2 = \frac{n(n^2-1)}{12} \#(10)$$

Now we can use these results to obtain a much simpler version of r_s as described below:

$$r_s = \frac{\sum_{i=1}^{n} \left(R(X_i) - \frac{n+1}{2}\right)\left(R(Y_i) - \frac{n+1}{2}\right)}{\frac{n(n^2-1)}{12}} \#(11)$$

Yet, there is another version of the Spearman's correlation coefficient was developed in the literature or used in textbooks of the correlation coefficients. Let $D_i = R(X_i) - R(Y_i)$ for $i = 1, \dots, n$. It is also true that

$$D_i = \left[R(X_i) - \overline{R(X_i)}\right] - \left[R(Y_i) - \overline{R(Y_i)}\right] \#(12)$$

since $\overline{R(X_i)} = \overline{R(Y_i)} = \frac{n+1}{2}$. Thus, it can be written that

$$\sum_{i=1}^{n} D_i^2 = \sum_{i=1}^{n} \left[R(X_i) - \overline{R(X_i)}\right] - \left[R(Y_i) - \overline{R(Y_i)}\right]^2 \#(13)$$

$$= \sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)^2 + \sum_{i=1}^{n} \left(R(Y_i) - \overline{R(Y_i)}\right)^2 \#(13)$$

$$-2\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)\left(R(Y_i) - \overline{R(Y_i)}\right) \#(13)$$

$$\sum_{i=1}^{n} D_i^2 = \frac{n(n^2-1)}{12} + \frac{n(n^2-1)}{12} - 2\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)\left(R(Y_i) - \overline{R(Y_i)}\right) \#(13)$$

If the last expression on the right of the equation is isolated, we can obtain

$$\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)\left(R(Y_i) - \overline{R(Y_i)}\right) = \frac{n(n^2-1)}{12} - \frac{\sum_{i=1}^{n} D_i^2}{2} \#(14)$$

So the numerator (covariance) is replaced with the above result and the denominator terms (square root of variances) are replaced with $\sum_{i=1}^{n} \left(R(X_i) - \overline{R(X_i)}\right)^2 = \frac{n(n^2-1)}{12}$ then a further simplification gives us another definition of r_s which can algebraically be reduced to

$$r_s = 1 - \frac{6\sum_{i=1}^{n} D_i^2}{n(n^2-1)} \#(15)$$

Where $D_i = R(X_i) - R(Y_i)$. If some ties exist in the samples, a small estimation error will occur in the last formula. With a few ties can be tolerated without a significant difference. The average method can be used to break the ties if necessary. All three versions of r_s provide the same results, but each has somewhat different computational difficulties.

3 SPEARMAN'S RANK CORRELATION BASED ON GENERAL SCORE FUNCTIONS

Spearman's r_s estimator of the correlation parameter ρ can be computed by using a general score function $\varphi(u)$ where the score function is the nondecreasing function defined on the interval (0,1) such that $\int_0^1 \varphi^2(u)du < \infty$. Hettmansperger [illegible] and Hettmansperger and McKean [illegible] defines the correlation coefficient estimator based on a general score function $\varphi(u)$ as

$$r_a = \frac{1}{s_a^2}\sum_{i=1}^{n} a\big(R(X_i)\big)a\big(R(Y_i)\big)\#(16)$$

Where $a(i) = \varphi(i/n+1)$ and $s_a^2 = \sum_{i=1}^{n} a^2(i)$ for $i = 1, \dots, n$. The score function $\varphi(u)$ can be standardized so that it satisfies $\int_0^1 \varphi(u)du = 0$ and $\int_0^1 \varphi^2(u)du = 1$. Moreover, $E(r_a)$ and $\text{Var}(r_a)$ can be derived with the following theorem.

Theorem 1. Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be a random sample from a bivariate continuous distribution function $F(X, Y)$. Under the null hypothesis of $H_0: \rho = 0$ ($F(X,Y) = F_x(X)F_y(Y)$ or independence of X and Y), the test statistics r_a satisfies the following properties: $E[r_a] = 0$ and $\text{Var}[r_a] = 1/(n-1)$

Proof. We made this assumption that the scores are generated with $a(i) = \varphi(i/n+1)$ for $i = 1, \dots, n$ and $a(1) \le a(2) \le \cdots \le a(n)$ based on the assumptions that the score function $\varphi(u)$ is nondecreasing. The score function is also centered so that $\int_0^1 \varphi(u)du = 0$ and $\int_0^1 \varphi^2(u)du = 1$. By using the Riemann sum approximation on the integral, it can be shown that $\int_0^1 \varphi(u)du \doteq \sum_{i=1}^{n} \varphi\left(i/(n+1)\right)\frac{1}{n} = \sum_{i=1}^{n} a(i)\frac{1}{n} \approx 0$. Also, $\int_0^1 \varphi^2(u)du \doteq \sum_{i=1}^{n} \varphi^2(i/(n+1))\frac{1}{n} = \sum_{i=1}^{n} a^2(i)\frac{1}{n} = \frac{s_a^2}{n} \approx 1$ can be approximated in a similar fashion.

This shows that the ranks, $R(X_i)$ (also $R(Y_i)$) is uniformly distributed for the integers of $i = 1, \dots, n$. Thus, $P[(R(X_i) = k] = 1/n$, where k is the rank of ith observation. So each rank of X_i is equally likely distributed with a probability of $1/n$. Therefore, from the expected value of $a\big(R(X_i)\big)$ it can be found that $E\left[a\big(R(X_i)\big)\right] = \sum_{i=1}^{n} a(i)\frac{1}{n} = 0$ since $\sum_{i=1}^{n} a\big(R(X_i)\big) = 0$. Similarly, $E\left[a\big(R(Y_i)\big)\right] = \sum_{i=1}^{n} a(i)\frac{1}{n} = 0$. These above results are substituted into the expression of r_a as defined in (16) which shows $E[r_a] = 0$.

To show part, the variance definition of $V[r_a] = E[(r_a)^2] - (E[r_a])^2$ can be used for the estimator r_a. In the first part of the proof it has been shown that $E[r_a] = 0$. Thus, the variance expression can be simplified to $V[r_a] = E[(r_a)^2]$.

Assuming under $H_0: \rho = 0$,

$$E[(r_a)^2] = E\left\{\left[\frac{1}{s_a^2}\sum_{i=1}^{n} a\big(R(X_i)\big)a\big(R(Y_i)\big)\right]^2\right\}$$

$$= \frac{1}{s_a^4}\sum_{i=1}^{n} E\big[a\big(R(X_i)\big)a\big(R(Y_i)\big)\big]\sum_{j=1}^{n} E\left[a\left(R\left(X_j\right)\right)a\left(R\left(Y_j\right)\right)\right]$$

$$= \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E\big[a\big(R(X_i)\big)a\big(R(Y_i)\big)\big]E\left[a\left(R\left(X_j\right)\right)a\left(R\left(Y_j\right)\right)\right]$$

$$= \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E\left[a\big(R(X_i)\big)a\left(R\left(X_j\right)\right)\right]E\left[a\big(R(Y_i)\big)a\left(R\left(Y_j\right)\right)\right]$$

To find the result of the expectations in the last expression, there are two cases ($i = j$ and $i \neq j$) that must be considered separately. For $i = j$,

$$E\left[a\big(R(X_i)\big)a\left(R\left(X_j\right)\right)\right] = E[a^2(X_i)] = \sum_{i=1}^{n} a^2(X_i)\frac{1}{n} = \frac{1}{n}s_a^2$$

Similarly, it can be found that

$$E\left[a\big(R(Y_i)\big)a\left(R\left(Y_j\right)\right)\right] = \frac{1}{n}s_a^2$$

For $i \neq j$, it can be written,

$$E\left[a\big(R(X_i)\big)a\left(R\left(X_j\right)\right)\right] = \sum_{i=1}^{n}\sum_{j=1}^{n} a\big(R(X_i)\big)a\left(R\left(X_j\right)\right)\frac{1}{n(n-1)}$$

$$= \frac{1}{n(n-1)}s_a^2$$

The last result can be applied to $E\left[a\big(R(Y_i)\big)a\left(R\left(Y_j\right)\right)\right] = \frac{1}{n(n-1)}s_a^2$. [illegible] cases of $i = j$ and $i \neq j$.

$$E[(r_a)^2] = \frac{1}{s_a^4}\sum_{i=1}^{n}\sum_{j=1}^{n} E\left[a(R(X_i))a\left(R(X_j)\right)\right]E\left[a(R(Y_i))a\left(R(Y_j)\right)\right]$$

$$= \frac{1}{s_a^4}\left\{\sum_{i=j=1}^{n} E[a^2(X_i)]E[a^2(Y_i)] + \sum_{i\neq j}\sum_{i\neq j} E\left[a(R(X_i))a\left(R(X_j)\right)\right]E\left[a(R(Y_i))a\left(R(Y_j)\right)\right]\right\}$$

$$= \frac{1}{s_a^4}\left\{\sum_{i=j=1}^{n} \frac{1}{n}s_a^2\frac{1}{n}s_a^2 + \sum_{i\neq j}\frac{1}{n(n-1)}s_a^2\frac{1}{n(n-1)}s_a^2\right.$$

$$= \frac{1}{s_a^4}\left[\frac{ns_a^4}{n^2} + n(n-1)\frac{s_a^4}{n^2(n-1)^2}\right]$$

$$= \frac{1}{s_a^4}\left[\frac{s_a^4}{n} + \frac{s_a^4}{n(n-1)}\right]$$

$$= \frac{1}{s_a^4}\left[\frac{(n-1)s_a^4}{n(n-1)} + \frac{s_a^4}{n(n-1)}\right]$$

$$= \frac{1}{s_a^4}\frac{s_a^4}{n-1}$$

$$E[(r_a)^2] = \frac{1}{n-1}$$

Thus, using the results of $E[r_a] = 0$ and $E[(r_a)^2] = \frac{1}{n-1}$ as shown above, we can prove that $V[r_a] = \frac{1}{n-1}$. The finite sample distribution of r_a is not easy to find, thus we can use the classical central limit theorem, $z_a = \frac{r_a}{\sqrt{1/(n-1)}} = r_a\sqrt{n-1} \to N(0,1)$ as n goes to ∞. For an asymptotic large sample α level test, we can then define that reject $H_0: \rho = 0$ if $|z_a| > z_{\alpha/2}$.

For example, if $\varphi(u)$ can be taken as $\varphi(u) = \sqrt{12}(u - 1/2)$ which is called the Wilcoxon's linear score function. It can be shown with the following theorem that Spearman's rank correlation (r_s) is equal to the r_a which is defined in Eq(16) if Wilcoxon's linear score function is used to estimate the correlation coefficient of r_a.

Theorem 2. Let $(X_1, Y_1), \dots, (X_n, Y_n)$ be a random sample from a bivariate continuous distribution function $F(X, Y)$. Also let $\varphi(u) = \sqrt{12}(u - 1/2)$ which is called the Wilcoxon's linear score function. Under the null hypothesis of $H_0: \rho = 0 (F(X,Y) = F_x(X)F_y(Y)$ or independence of X and Y) then the test statistics based on Wilcoxon's rank score r_a is equal to the Spearman's r_s with the same variance and the expected value as defined in theorem 1.

Proof. First, recall that

$$a(i) = \varphi[(i/n + 1)] = \sqrt{12}(i/(n+1) - 1/2)\#(17)$$

which is called Wilcoxon's score function. Also, according to [illegible] s_a^2 is defined as

$$
\begin{aligned}
s_a^2 &= \sum_{i=1}^{n} a^2(i) \\
&= \sum_{i=1}^{n} \{\sqrt{12}[i/(n+1) - 1/2]\}^2 \\
&= \frac{12}{(n+1)^2}\left[\sum_{i=1}^{n} i^2 - (n+1)\sum_{i=1}^{n} i + \frac{n(n+1)^2}{4}\right] \\
&= \frac{n(n-1)}{n+1}
\end{aligned}
$$

Now we also let

$$a\big(R(X_i)\big) = \sqrt{12}[R(X_i)/(n+1) - 1/2] \#(18)$$

and similarly

$$a\big(R(Y_i)\big) = \sqrt{12}[R(Y_i)/(n+1) - 1/2] \#(19)$$

The [illegible] results [illegible] the r_a which is defined [illegible]. [illegible] can find

$$r_a = \frac{\sum_{i=1}^{n} \left[(R(X_i) - (n+1)/2)(R(Y_i) - (n+1)/2)\right]}{n(n^2-1)/12} \#(20)$$

which is also defined as r_s [illegible]. [illegible] result of the theorem [illegible] proves [illegible] that r_s can also be estimated using a score based methods such as Wilcoxon's score function.

Yet there is another nonparametric alternative to estimate ρ which is called Kendall's τ estimator [illegible]. It uses the ranks of the observations [illegible] counts the concordant and discordant pairs of the observations in the pairs of random samples. An estimator of Kendall's τ can be derived [illegible]

$$r = \frac{(n_c - n_d)}{(n(n-1))/2} \#(21)$$

where n_c shows the number of concordant pairs and n_d shows the number of discordant pairs.

If we compare Pearson against [illegible] and [illegible] estimators, Pearson's correlation coefficient measures how well the linear relationship between two variables are. In reality, there could be other ways that two variables can be correlated such as monotonic relationship which does not show relationship with a straight line. Monotonic relationship between two variables can show exponential or logistic distributions such as when one variable increases the other increases or decreases consistently. Spearman and Kendall's correlation coefficients are basically a non-parametric way to estimate the monotonic relationship between two random variables with a continuous bivariate distribution function of $F(X,Y)$.

The Spearman correlation test assumes only that the data are a random sample. Spearman's correlation is calculated on the ranks of the observations and will therefore work with data that can be ranked, including ordinal, interval or ratio data. Because it is based on ranks, it is less sensitive to outliers than the Pearson correlation estimator. It is sometimes used to evaluate a correlation when outliers are present.

4 PROPOSED SMOOTHED RANKS BASED CORRELATION ESTIMATOR

First recall that the general score function is defined as

$$a(i) = \varphi[(i/n+1)] = \sqrt{12}(i/(n+1) - 1/2)$$

Which is based on the Wilcoxon's linear score function. When using the regular ranks we can write the score function as

$$a\big(R(X_i)\big) = \sqrt{12}[R(X_i)/(n+1) - 1/2]\#(22)$$

and similarly

$$a\big(R(Y_i)\big) = \sqrt{12}[R(Y_i)/(n+1) - 1/2]\#(23)$$

As it was shown in Theorem 1 and Theorem 2 the r_a is equal to the Spearman's r_s as defined in Eq(15). The proposed method replaces $R(X_i)$ and $R(Y_i)$ with a smoothed ranks and call them $\widehat{R(X_i)}$ and $\widehat{R(Y_j)}$, respectively. The proposed approach and the method described below has been provided [illegible].

In order to generate a rank set of a random sample of $X_1, X_2, \ldots, X_n$, the empirical cdf of the random sample is defined as

$$F_n(x) = \frac{\#\{X_j \le x\}}{n} = \frac{1}{n}\sum_{j=1}^{n} \mathbf{I}\left(X_j \le x\right)\#(24)$$

where $\mathbf{I}$ is called the indicator function. This is a stepwise discrete function at each X_i. Then the ranks of a random sample can be expressed as

$$R(X_i) = nF_n(X_i) = \sum_{j=1}^{n} \mathbf{I}\left(X_j \le X_i\right)\#(25)$$

where $R(X_i)$ is the rank of the i th observation. In a similar fashion,

$$R(Y_i) = nF_n(Y_i) = \sum_{j=1}^{n} \mathbf{I}\left(Y_j \le Y_i\right)\#(26)$$

We now consider a smooth approximation to the indicator function by replacing it with a continuous distribution function $H(x/h)$, where the scale parameter h is called the bandwidth or smoothing parameter. Kernel smoothing applications are such that h approaches zero as the sample size increases. The distribution function $H(x)$ is chosen to be a continuous non-decreasing and also to be the function that generates smoothed ranks as described below. The resulting smoothed empirical function can be written as

$$F_s(t) = \frac{1}{n}\sum_{j=1}^{n} H\left(\frac{t - X_j}{h}\right)\#(27)$$

This smoothing result can be used to define the smoothed ranks

$$\widehat{R(X_i)} = nF_s(X_i) = \sum_{j=1}^{n} H\left(\frac{X_i - X_j}{h}\right)\#(28)$$

where the notation $\widehat{R(X_i)}$ indicates the smoothed rank of the observation X_i. It is important to mention that the purpose of smoothing is to avoid estimating a density function, but to use $H(x)$ function as a smooth approximation of the indicator function I. Therefore, the smoothed ranks can be generated from the $F_s(t)$ function. Moreover, it should be noted that when $x_i > X_j$, $H\left(\frac{x_i - X_j}{h}\right) \to 1$ when h approaches zero as n gets large.

Similarly, when $x_i < X_j, H\left(\frac{x_i - X_j}{h}\right) \to 0$ when h approaches zero as n gets large.

Also, it should be noted that

$$H_n\left(X_{(i)}\right) = \frac{i-1}{n} + \frac{X_{(i)} - V_i}{n(V_{i+1} - V_i)}\#(29)$$

Where $X_{(i)}$ is the i th order observation and V_i is the midpoint between X_i and X_{i+1}. Also, $V_1 = X_1 - (V_2 - X_1)$ and $V_{n+1} = X_n + (Z_n - V_n)$.

As a result of the smoothing and replacing the indicator function with $H(x/h)$, the components of the smoothed ranks using smoothed distribution function of $F_s(X_i)$ are defined as

$$\widehat{R(X_\iota)} = nF_s(X_i)\#(30)$$

and

$$\widehat{R(Y_\iota)} = nF_s(Y_i)\#(31)$$

Then using these smoothed ranks we redefine the smoothed score function

$$a\left(\widehat{R(X_\iota)}\right) = \sqrt{12}\left[\widehat{R(X_\iota)}/(n+1) - 1/2\right]\#(32)$$

and similarly,

$$a\left(\widehat{R(Y_\iota)}\right) = \sqrt{12}\left[\widehat{R(Y_\iota)}/(n+1) - 1/2\right]\#(33)$$

We ca[illegible] the new scores into r_a as def[illegible] to derive the new version of Spearman's correlation estimator based on the smoothed Wilcoxon score function:

$$r_{sa} = \frac{1}{s_a^2}\sum_{i=1}^{n} a\left(\widehat{R(X_\iota)}\right)a\left(\widehat{R(Y_\iota)}\right)\#(34)$$

Where $s_a^2 = \sum_{i=1}^{n} a^2(i) = \frac{n(n-1)}{n+1}$ [illegible] Wilcoxon's score function. A further simplification of the above results gives

$$r_{sa} = \frac{\sum_{i=1}^{n}\left[\left(\widehat{R(X_\iota)} - (n+1)/2\right)\left(\widehat{R(Y_\iota)} - (n+1)/2\right)\right]}{n(n^2-1)/12}\#(35)$$

Moreover, we can replace $R(X_i)/(n+1)=\frac{n}{n+1}F_n(X_i)$ by $H_n\left(X_{(i)}\right)$ and $R(Y_i)/(n+1)=\frac{n}{n+1}F_n(Y_i)$ by $H_n\left(X_{(i)}\right)$ and $H_n\left(Y_{(i)}\right)$ respectively. Then we get

$$a\left(\widehat{R(X_\iota)}\right)=\sqrt{12}\left[\widehat{H_n(X_\iota)}-1/2\right]\#(36)$$

Similarly

$$a\left(\widehat{R(Y_\iota)}\right)=\sqrt{12}\left[\widehat{H_n(Y_\iota)}-1/2\right]\#(37)$$

Hence we can find this result to r_a and form the new version of the Spearman correlation coefficient estimator based on kernel function

$$r_{sa}=\frac{\sum_{i=1}^{n}\left[\left(H_n\left(X_{(i)}\right)-(n+1)/2\right)\left(H_n\left(Y_{(i)}\right)-(n+1)/2\right)\right]}{n(n^2-1)/12}\#(38)$$

4.1 BANDWIDTH SELECTION

It is well known that the selection of bandwidth h is more important than the shape of the kernel function $H(x)$ as stated in [illegible]. As a result, an appropriate selection of a bandwidth h must be considered in order to obtain a smooth approximation of the indicator function. There are many bandwidth h options available in the smoothing applications such as bandwidths suggested by [illegible] and [illegible] are options to be considered because of their computational performance and also availability in the software. The bandwidth suggested by [illegible] is also called the rule of thumb bandwidth approach in the literature. It has a numerical formula that equals to $h=0.9\hat{\sigma}n^{-0.20}$ where $\hat{\sigma}=\min\{s, IQR/1.349\}$. [illegible] suggests a least squares cross validation approach and it is also called an unbiased cross validation smoother in the literature. [illegible] approach is also commonly used and considered to be a good performer as suggested by [illegible]. The last two options have no closed form formulas. [illegible] used these three bandwidth options for the smooth version of the Kolmogorov-Smirnov statistic and has shown all three bandwidth options performed similarly in estimating the shift parameter for location problems. Moreover, we also considered the smoothing parameter $h=\hat{\sigma}n^{-0.26}$ as suggested by [illegible]. It also satisfies $nh\to\infty$ and $nh^4\to 0$ conditions in order to have an optimal rate of convergence. The $\hat{\sigma}$ is the estimated pooled standard deviation from the data. For robustness purposes we consider using MAD (Median Absolute Deviation) to estimate σ instead of a regular standard deviation approach. No parametric study of these four different adjustment parameters will be discussed in detail later in Section.

5 MONTE CARLO SIMULATION STUDY

In this section, Monte Carlo simulation was performed and the mean square error (MSE) of the ρ parameter for each estimator r is computed. The MSE is formulated as $MSE(r) = (r-\rho)^2 + V(r)$ which is found as average of bias ^{2} and the variance of correlation estimator r. First, two random samples of size $n = 50$ are generated from a bivariate normal distribution with parameters $\mu_1 = 2, \mu_2 = 4, \sigma_1 = 1, \sigma_2 = 1$ and with the correlation parameter ρ which is set to the sequence of $\rho = \{0,0.02,0.04,\dots,1\}$.

For each parameter value of the ρ, Person's r_p, Spearman's r_s, Kendall's r_k and the proposed smoothed ranked based r_{sa} estimator are used for estimating the MSE values. The figure shows the result of the bivariate normal distribution which is defined below

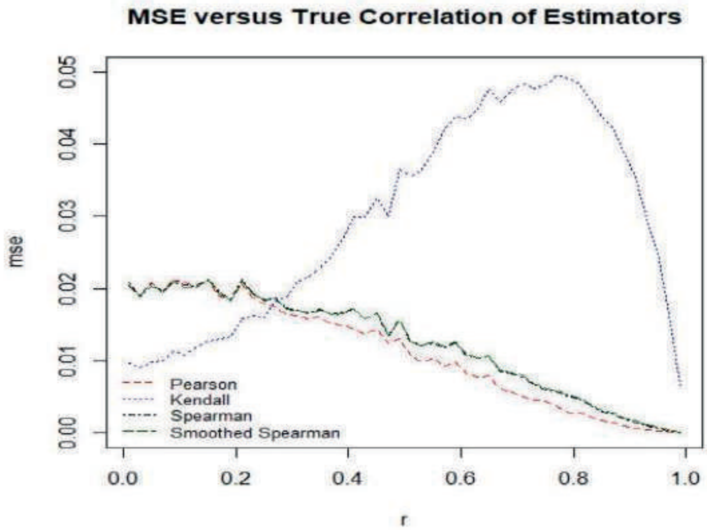
$$f_{XY}(x,y) = \frac{1}{2\pi\sqrt{1-\rho^2}} \exp\left\{-\frac{1}{2(1-\rho^2)}[x^2 - 2\rho xy + y^2]\right\}$$

where $-1 \le \rho \le 1$.

The figure is generated from the bivariate exponential distribution also known as Farlie-Gumbel-Morgenstern distribution is defined

$$F(x,y) = F_1(x)F_2(y)[1 + \rho(1 - F_1(x))(1 - F_2(y))]\#(39)$$

for $x \ge 0$ and $y \ge 0$ and the marginal distribution functions F_1 and F_2 are exponential with scale parameters θ_1 and θ_2 and correlation parameter ρ where $-1 \le \rho \le 1$. This formulation follows that the marginal distributions of X and Y are exponential with parameters α and β.



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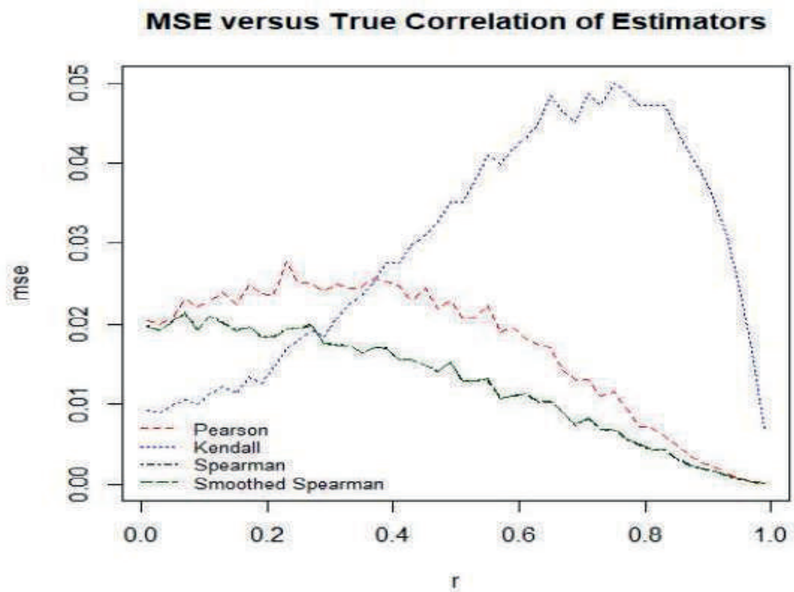

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approaches [illegible]. On the other hand, Pearson's r_p performs the best overall [illegible] the ρ approaches [illegible]. Moreover, the proposed smoothed rank and Spearman estimates and [illegible] estimates each [illegible] overlaps. Therefore, if we compare the all estimators, Pearson's r_p performs the best under the bivariate normal, whereas Spearman and the smoothed ranked estimates perform similarly and looks very stable. On the other hand, Kendall's r_k is the worst performer as shown in the Figure [illegible]. As the Figure [illegible] shows that if we assume that Y are coming from a bivariate exponential distribution, then the proposed smoothed rank and Spearman's correlation estimates performs the best overall. Pearson's [illegible] values are higher than Spearman and the smoothed rank based estimates under bivariate exponential case. This is expected since Pearson is known to be a good estimator under normal or linear relationships. Kendall's r_k shows a [illegible] as shown previously under the bivariate normal case.

In the second part of the simulation, the relative efficiencies of the smoothed ranked r_{sa} with respect to the other estimators are compared based on the ratios of the estimated [illegible]. The results are presented in the Table [illegible]. The most obvious result is that the proposed smoothed rank's r_{sa} estimator performs very similar to the Spearman's r_s methods since the relative efficiencies are very close to [illegible], which is expected since the same [illegible] similar to the Figure [illegible]. Pearson shows superior power over the other estimators if the underlying distribution is bivariate normal since the relative efficiency results as ρ goes to [illegible]. Between smoothed and Kendall's estimator, smoothed rank based estimator has increasing relative efficiencies as ρ increases.

Table 1: Relative efficiency rates of the ρ estimators based on [illegible] under bivariate normal

Correlation (ρ)	Pearson/Smooth	Kendall/Smooth	Spearman/Smooth	Pearson/Spearman
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

A [illegible] are [illegible].

Table 2: [illegible] of [illegible] ρ [illegible]

[illegible] ρ [illegible]	Pears[illegible]	[illegible]	[illegible]	Pears[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

As [illegible] see [illegible] r_{sa} [illegible] r_s [illegible] as ρ [illegible]. [illegible] Pearso[illegible] ρ [illegible] 0.50 − 0.75. [illegible] as [illegible] as ρ [illegible].

6 CONCLUSIONS

[illegible] (ρ) [illegible] assesses [illegible] of [illegible]. [illegible] ρ [illegible] as

$$\rho = 1 - \frac{6 \sum D_i^2}{n(n^2 - 1)} \#(40)$$

[illegible] D_i [illegible] of [illegible] x [illegible] y [illegible] n [illegible] of [illegible] are [illegible] or [illegible] of [illegible] score [illegible] of [illegible] as [illegible] of [illegible] or [illegible].

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7 REFERENCES

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Conflict of Interest

Authors declare that there is no conflict of interest in the preparation of this article and the use of all datasets.

Chapter 9

The Impact of Climate Variables on Wheat Yield in Turkey

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Gülistan Erdal[2]

Abstract

In recent years, global warming and climate change have been affecting agricultural production both directly and indirectly. Rising temperatures, changing precipitation patterns, and extreme weather events have significant impacts on the yield of many crops. In Turkey, which is located in a semi-arid climate zone, this variability becomes even more pronounced. Staple cereal crops such as wheat are particularly sensitive to fluctuations in temperature and precipitation, and climatic anomalies occurring during critical growth stages can lead to substantial yield losses.This study aims to examine the effects of climate variables on wheat yield in Turkey. The dataset covers the period from 2004 to 2024. Wheat yield (Y, kg/ha) was used as the dependent variable, while climate-related independent variables (Xi) included temperature (°C), average precipitation (mm), the Southern Oscillation Index (SOI), and the Oceanic Niño Index (ONI). The SOI and ONI indicators employed in the study provide important parameters for understanding the impact of large-scale climatic oscillations on crop yields. In this context, ocean-atmosphere interactions such as the El Niño–Southern Oscillation (ENSO), which operate on a global scale, create significant regional and temporal variability in agricultural production.The results of the multiple linear regression analysis show that the model is statistically significant, with a high coefficient of determination (R^2 = 64.3%). Among the climate variables, temperature and SOI were found to be statistically significant. In particular, SOI was determined to have a strong and positive effect on wheat yield. The analysis results reveal that climate indicators play a critical role

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in wheat productivity. The findings of this study offer valuable insights for the development of sustainable agricultural policies and the formulation of adaptation strategies in response to climate change.

Introduction

Wheat is both a fundamental food staple and a strategic agricultural commodity. It forms the basis of food security, especially in developing countries. Beyond being merely a food product, wheat holds a critical position in countries' agricultural policies, trade balances, and rural development strategies. Globally, wheat production in 2024 reached approximately 785 million tons, with an average yield of 3.7 tons per hectare (FAO, 2025). In Turkey, wheat production in 2024 was about 20.8 million tons, with a yield of 2.4 tons/ha. Wheat production is highly sensitive to climatic conditions. In particular, increases in temperature, drought, frost events, and changes in precipitation patterns directly affect both yield and quality. Turkey, located in a semi-arid climate zone, is among the countries with a high potential to be affected by climate change, a situation that becomes especially apparent in the production of strategically important cereal crops such as wheat. Wheat not only constitutes one of the fundamental elements of Turkey's food security but also plays a critical role in the sustainability of rural economies. However, this crop is very sensitive to climatic variables such as temperature and precipitation during its growth and development stages (İm et al., 2020). Indeed, studies have shown that increases in temperature and irregular precipitation patterns can negatively impact wheat yields (Asseng et al., 2015; Zampieri et al., 2017). Moreover, not only local climatic variables but also large-scale ocean-atmosphere interactions can affect agricultural production. Large-scale climatic oscillations such as the El Niño–Southern Oscillation (ENSO) alter precipitation and temperature regimes, indirectly influencing regional productivity (Rosenzweig et al., 2001).

Global warming and climate change have recently become one of the most significant environmental issues directly and indirectly affecting the agricultural sector. Rising temperatures, changing precipitation patterns, and the increasing frequency of extreme weather events pose serious threats to the productivity of climate-sensitive crops (Lobell et al., 2011; Wheeler & von Braun, 2013). This study focuses on examining the impacts of climatic variables on wheat yield in Turkey. Specifically, the roles of temperature, precipitation, the Southern Oscillation Index (SOI), and the Oceanic Niño Index (ONI) on wheat yield between 2004 and 2024 have been analyzed using a multiple linear regression model. The primary aim of the study is to analyze the relationship between wheat yield and selected climatic variables in Turkey during the 2004–2024 period. The research seeks to reveal the effects of both local and global climatic indicators such as temperature, precipitation, SOI, and ONI on wheat productivity. The findings are expected to contribute to more accurate assessments of climate risks in agricultural production planning, the development of adaptation policies for climate change, and sustainable agricultural strategies. The conceptual framework of the study revolves around core concepts such as climate change, agricultural production, productivity, climatic variability, and large-scale climatic oscillations.In this context, productivity (kg/ha) represents the amount of product obtained per unit area, with climatic factors such as temperature, precipitation amount and distribution, and extreme weather events being the primary components affecting this value. Furthermore, large-scale ocean-atmosphere interactions at the global level, particularly phenomena such as the El Niño–Southern Oscillation (ENSO), indirectly affect regional climate conditions, causing significant fluctuations in agricultural production. The two main indicators of ENSO—the Southern Oscillation Index (SOI) and the Oceanic Niño Index (ONI)—are standard climatic indices used to analyze the direction and intensity of these oscillations. This study empirically evaluates the effects of these concepts, integrating both local climatic variables and global climate indices, on wheat yield.

Materials and Methods

Data

This study aims to investigate the climatic and structural factors affecting wheat yield in Turkey. Annual data from the period 2004–2024 were used in the analysis. The dependent variable considered was wheat yield per hectare (kg/ha). The independent variables were defined as follows:

- Average temperature (°C),
- Annual average precipitation (mm),
- Southern Oscillation Index (SOI),
- Oceanic Niño Index (ONI),
- Number of tractors (units),
- Number of combine harvesters (units).

Wheat yield, tractor, and combine harvester data were obtained from the Turkish Statistical Institute (TurkStat), while climate variables were sourced from the General Directorate of Meteorology (MGM) and the National

Oceanic and Atmospheric Administration (NOAA). The number of tractors and combine harvesters represent the level of mechanization in agricultural production and were included in the model as structural indicators that may affect production capacity. The SOI and ONI indices were used to analyze the effects of the global El Niño–Southern Oscillation (ENSO) events on agricultural production in Turkey.

Method

The effects of climatic and structural factors on wheat yield were evaluated using multiple linear regression analysis. The analyses were conducted using the SPSS 26.0 software package (Field, 2013). The overall significance of the model was assessed by the F-test, while the individual contributions of the variables were tested with t-tests. The explanatory power of the model was measured by the coefficient of determination (R^2), which was calculated as 64.3%. The dependent variable in the regression model was wheat yield, and the independent variables included:

- Temperature (°C)
- Average precipitation (mm)
- Southern Oscillation Index (SOI)
- Oceanic Niño Index (ONI)
- Number of tractors (units)
- Number of combine harvesters (units)

The model tests the combined effect of climatic conditions and the level of agricultural mechanization on yield. In the regression table, the overall significance of the model is checked with the F-test. If the p-value is less than 0.05, the model is considered significant. The R^2 value indicates how much of the variation in the dependent variable is explained by the independent variables (Gujarati & Porter, 2009). The coefficients (B) of the independent variables explain their effect on yield. Variables with positive coefficients increase yield, while those with negative coefficients decrease it. Variables with a p-value less than 0.05 are statistically significant (Wooldridge, 2016). The regression model is defined as follows:

$$Yt = \beta0 + \beta1 \cdot Temperature_t + \beta2 \cdot Precıpıtıon_t, + \beta3 \cdot SOIt + \beta4 \cdot ONIt + \beta5 \cdot Tractor_t + \beta6 Harvester_t + \varepsilon t$$

Where:

- Yt = wheat yield in year (kg/ha),

- βi= coefficients estimated by the model,
- εt = error term.

The assumptions of regression, including normality of data, multicollinearity, autocorrelation, and homoscedasticity, were tested. Variable transformations and outlier analyses were performed when necessary.

RESULTS

Table 1. Wheat Production and Yield Development in the World and Turkey Between 2004 and 2024

Year	World Production (million tons)	World Yield (tons/ha)	Turkey Production (million tons)	Turkey Yield (tons/ha)
2004	596	2.7	21.0	2.4
2005	621	2.8	20.5	2.3
2006	589	2.7	20.0	2.2
2007	610	2.8	17.2	2.0
2008	683	3.0	17.8	2.1
2009	681	3.0	20.06	2.4
2010	651	2.9	19.7	2.3
2011	697	3.1	21.8	2.5
2012	655	3.0	20.1	2.3
2013	717	3.2	22.1	2.6
2014	728	3.3	19.0	2.2
2015	736	3.4	22.6	2.7
2016	757	3.4	20.6	2.5
2017	762	3.3	21.5	2.6
2018	731	3.4	20.0	2.4
2019	759	3.5	20.5	2.5
2020	773	3.6	20.5	2.5
2021	780	3.7	17.7	2.2
2022	789	3.7	19.8	2.4
2023	785	3.7	21.0	2.5
2024	791	3.7	20.8	2.4

Between 2004 and 2024, a continuous increase in global wheat production and productivity has been observed. World wheat production rose from 596 million tons in 2004 to 791 million tons in 2024, while yield increased from 2.7 tons per hectare to 3.7 tons per hectare during the same period. This indicates significant advancements worldwide in areas such as agricultural technologies, irrigation systems, climate adaptation, and seed quality.

In Turkey, however, there have been considerable fluctuations in production volumes during the same period. Especially in 2007 and 2021, production significantly declined to 17.2 million tons and 17.7 million tons, respectively. Turkey's wheat yield remained stagnant at 2.4 tons/ha in both 2004 and 2024. This situation reveals that agricultural productivity in Turkey lags behind the world average and needs to be supported by more sustainable, technology-based policies. Compared to the world, Turkey has experienced slow growth in yield and appears to be more sensitive to external factors such as climate conditions in terms of production. This picture also highlights the need for modernization in Turkish agriculture and its vulnerability to climate change.

Table 2. Summary of Regression Model Results

Variables	**B**	**Std. Hata**	**Beta**	**t**	**p**	**Sig.**
Turkey Average Temperature (°C)	-270.63	102.89	-0.507	-2.630	.020*	,020
SOI (Southern Oscillation Index)	260.20	113.04	0.433	2.302	.037*	,037
Number of Combine Harvesters (units)	0.099	0.030	0.948	3.325	.005**	,005

Model Statistics:

$R = .802$, $R^2 = .643$, Adjusted $R^2 = .490$,

$F(6, 14) = 4.20$, $p = .013$

Note: *$p < .05$, **$p < .01$. Dependent variable: Wheat yield in Turkey (kg/ha).

The R value (0.802) indicates a strong linear relationship between the dependent variable and the independent variables.

R Square (0.643) shows that the model explains 64.3% of the total variance, indicating that approximately two-thirds of the variability in wheat yield is explained by the model.

Adjusted R Square (0.490) accounts for the number of independent variables and shows that the explanatory power of the model decreases to 49.0%. This suggests that some variables may have limited contribution to the model.

Standard Error of the Estimate (198.823) represents the average size of the prediction errors.

Average Temperature in Turkey: The result is negative and significant, indicating that with a 1°C increase, yield decreases by 270 units, showing that temperature negatively affects yield.

SOI: Positive and significant, meaning that as the SOI (La Niña) effect increases, yield also increases.

Number of Combine Harvesters: Positive and strong result, indicating that as mechanization increases, yield significantly improves.

Results

his study examines the climatic and mechanical factors affecting agricultural productivity in Turkey through regression analysis. The findings reveal that agricultural productivity is significantly influenced by temperature, atmospheric circulation indices, and agricultural mechanization elements.

The first significant variable in the model is the average temperature in Turkey. The results indicate that increases in average temperature have a negative and statistically significant effect on agricultural productivity. This finding demonstrates that rising temperatures associated with climate change threaten agricultural production, and this threat is measurable in concrete terms. Accordingly, agricultural policies should focus on developing heat-resistant seed varieties, enhancing microclimatic measures, and strengthening irrigation infrastructure.Secondly, the positive and significant effect of global atmospheric variables such as the Southern Oscillation Index (SOI) is noteworthy. This suggests that Turkey's agricultural production is influenced by large-scale climate cycles. In this context, it is important to develop early warning systems for predicting global climate events such as El Niño and La Niña in agricultural production planning.

One of the most striking findings is the strong and positive impact of the number of combine harvesters on agricultural productivity. This indicates that increasing mechanization directly supports productivity. Facilitating access to harvesting machinery like combine harvesters reduces time loss during the production process and minimizes crop losses. Therefore, the widespread implementation of low-interest loan programs for agricultural machinery procurement, shared machinery parks, and regional equipment support programs is necessary.

Conversely, the lack of a significant effect of the number of tractors and precipitation on agricultural productivity suggests that considering these variables only quantitatively is insufficient. Particularly, including qualitative features such as tractor usage intensity, technological equipment, and maintenance status in the analysis, rather than merely the number of tractors, will yield more accurate results.

In conclusion, this study highlights the critical importance of developing climate change-adaptive agricultural strategies and strengthening mechanization infrastructure to increase agricultural productivity in Turkey. Future studies should include additional agricultural inputs such as soil fertility, fertilizer and pesticide use, and cropping patterns in the model, which will broaden the scope of policy recommendations.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

The authors contributed equally.

Chapter 10

Multi-Frequency and Multi-Protocol RFID Card Reader Device: Hardware and Application Design

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Muhammet Fatih Aslan[3]

Akif Durdu[4]

Abstract

In this study, a dual frequency supported Radio Frequency Identification (RFID) card reader device that can operate at both 125 kHz and 13.56 MHz is developed for use in access control systems. Most of the existing systems operate at a single frequency band and therefore offer limited card compatibility. But, the proposed design supports multiple RFID standards and offers a more flexible and user-friendly solution. The reader functioning at 125 kHz can successfully detect cards based on the EM4100 protocol, whereas at 13.56 MHz, it effectively communicates with MIFARE Classic, MIFARE Ultralight, and Near Field Communication (NFC)-compatible cards. In the hardware design, two RFID modules have been optimized and integrated under the control of a single microcontroller. The system is capable of automatically detecting the card's frequency, switching to the appropriate protocol, and performing secure identity verification processes. Additionally, the device has been designed to be resistant to electromagnetic interference and capable of operating inside a metal enclosure, taking into account harsh environmental conditions. Thus, it can also exhibit reliable performance in areas such as industrial facilities, parking systems and dense

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metal environments. Thanks to this innovative approach, not only has compatibility with existing card infrastructures been achieved, but also an important step has been taken for long-lasting and secure access control systems.

INTRODUCTION

Radio Frequency Identification (RFID) is a technology and system that uses radio waves (wireless) to remotely identify and track objects. RFID systems consist of tag, reader, and back-end database components. Data transmission is carried out via electromagnetic fields (Piramuthu, 2007). The tag contains object identification (ID) data, the reader reads this data, and the back-end database is used to store product information (Khan et al., 2024). In the early days, RFID was only used to basically identify an object (for example, to understand who a product is) or to track its location. However, later, chip RFID went far beyond optical barcode technology. Thanks to radio waves, out-of-sight communication was provided. Therefore, the object could be read even when it was in the box. In addition, unlike barcodes, RFID chips could store more information, such as production date, manufacturer, temperature history, and transportation information. In addition, RFID systems can automatically identify an object when it enters the reading area without the need for human intervention (Lasantha et al., 2023).

Today, RFID technology is widely and effectively used in different sectors. RFID has gained application in many sectors such as manufacturing (e.g. inventory tracking), logistics (Wanhua, 2020), healthcare (Cheng et al., 2021), retail (e.g. stock tracking), construction, transportation systems (Casella et al., 2022). It offers a contactless, fast, and reliable solution, especially in security-oriented applications such as access control. Its main advantages include automation capability, multiple tag recognition, and reliable performance even in harsh environmental conditions. For example, it can identify many RFID tags simultaneously in fast-flowing production lines or at points with busy passages. In addition, RFID tags that are resistant to harsh environmental conditions such as high temperature, humidity, dust, and impact offer suitable solutions for industrial environments (Zhao et al., 2022).

RFID systems are divided into three basic groups according to their operating frequencies: Low Frequency (LF, 125–134 kHz), High Frequency (HF, 13.56 MHz), and Ultra High Frequency (UHF, 860–930 MHz). Generally, a certain type of passive RFID tag can only be read by a reader of the same type. LF RFID systems have a short reading range but work more stably in challenging environments such as metal and liquid. HF systems have a short reading range of about 12 cm and are widely used in contactless smart cards and access systems. UHF RFID systems provide a reading range of up to 3 meters (Ashour et al., 2023).

In RFID systems, readers are generally designed to operate only at a certain frequency, which leads to significant limitations in system integration. Most RFID readers operate either with only LF, only HF or only UHF protocols. This requires users to install multiple systems and increases hardware costs and system complexity. Because multiple readers or complex access systems must be installed to read RFID tags operating at different frequencies. This poses a significant problem in terms of traceability and flexibility, especially in industrial applications. In addition, due to frequency dependency, systems may need to be adapted separately to legal frequency restrictions in different geographical regions. In this context, the demand for hybrid systems is increasing today, because installation, maintenance and user management become more efficient thanks to devices that can support both HF cards such as MIFARE and LF cards such as EM4100. In addition, the importance of such systems for Internet of Things (IoT) integrations, multi-device compatibility and comprehensive security systems is rapidly increasing (Hirvonen et al., 2008).

In the past literature, it has been frequently emphasized that RFID systems mostly operate at only a single frequency, which creates serious limitations on system flexibility and hardware costs. In this context, Cui et al. (2019) discussed the place of RFID technology in IoT systems in terms of energy efficiency, data transmission and sensor integration in a review study. In the study, they detailed the working principles of both HF and UHF RFID systems. They analyzed the impact of different antenna designs and RF energy harvesting methods on RFID sensor performance. The authors emphasized that RFID-based sensors are generally optimized only for HF or UHF systems, and multi-frequency systems are not yet widespread, which limits interoperability in applications. Similarly, Bajaj et al. (2024) detailed the limitations of modern RFID reader antenna technologies in a review article. They also presented research trends in the development of multi-frequency systems. The authors noted that LF, HF, and UHF bands each serve different application requirements, but unfortunately most current readers are optimized for only a single frequency band. Based on this, the authors clearly emphasized the need for multiple readers, especially in environments where a wide variety of cards may be used, such as logistics, supply chain, and healthcare systems. In a separate study, Fischer et al. (2019) developed a multi-frequency (868 MHz and 915 MHz) RFID reader system in the

UHF band. The system was configured on a software-defined radio (SDR) platform in accordance with international standards. With this system, the authors aimed to increase the reading success in multipath environments. Experimental results showed that successful reading was achieved from the same tag at both frequencies. Thanks to frequency diversity, blind spots known as "dead zones" were significantly reduced. This study technically demonstrated the advantages of multi-frequency systems in terms of reliability, especially in harsh industrial environments. Similarly, Bournine et al. (2015) developed a multi-frequency RFID reader prototype operating at HF and UHF frequencies. They also integrated this device into a cloud-based traceability system. In the designed system, they combined Adafruit PN532 HF module and HYM730 UHF module using Arduino Due platform. They managed the frequencies sequentially with microcontroller software. For a secure access model, after user authentication with HF tag, transactions were allowed via UHF. This study presented a multi-frequency RFID that supports different RFID protocols on the same device and can be integrated into traceability processes.

This study aims to develop a multi-frequency and multi-protocol RFID reader system that can operate at both LF (125 kHz) and HF (13.56 MHz) frequencies on a single device. The developed system can successfully identify EM4100 based low-frequency cards and MIFARE Classic, MIFARE Ultralight and NFC compatible high-frequency cards. Thus, this application aims to provide a flexible and multi-purpose solution in response to the limited frequency compatibility of traditional RFID readers, similar to previous studies. The reader side of the RFID allows two different RFID modules to be automatically controlled by a single microcontroller. In this way, users can perform access transactions without thinking about which card they use in the system. In addition, the device's electromagnetic interference-resistant structure and its ability to operate even in metal enclosures make it suitable for use in harsh industrial conditions.

MATERIAL AND METHODS

Hardware Components

In the proposed system, a dual-frequency RFID card reader architecture is developed to enable the detection of both low-frequency (125 kHz) and high-frequency (13.56 MHz) RFID cards. At the core of the system is the NUC1311L series microcontroller based on the Advanced RISC Machine (ARM) Cortex-M0 core. It also manages the communication with the peripheral components via Serial Peripheral Interface (SPI), Inter-

Integrated Circuit (I^2C) and Universal Asynchronous Receiver/Transmitter (UART) protocols.

For the LF section, an analog front end (AFE) circuit was designed according to the EM4100 protocol. The circuit consists of a resonant inductance-capacitance (LC) network operating at 125 kHz driven by a 50% duty cycle pulse-width modulation (PWM) signal generated by the microcontroller. A specially designed air core coil antenna optimized for high quality (Q) factor and mechanical stability was used. The multi-turn antenna design is shown in Figure 1. The inductance and resonant frequency were verified using an inductance-capacitance-resistance (LCR) meter and the antenna was tuned with a series capacitor. The magnetic signal induced in the LC circuit is passed through a low noise differential preamplifier, followed by an envelope detector and a comparator circuit to convert the signal to a digital format suitable for further processing. The digitized data is transferred to the digital input pin of the microcontroller and made available for timing analysis in the software layer.

For the HF section, a Texas Instruments TRF7970A RFID transceiver was used. This module supports International Organization for Standardization / International Electrotechnical Commission (ISO/IEC) 14443 and ISO/IEC 15693 standards, enabling communication with MIFARE Classic, MIFARE Ultralight, and near field communication (NFC) compatible cards. A planar spiral printed-circuit-board (PCB) antenna, designed according to calculation tools and radio frequency (RF) design guidelines, was directly interfaced with the TRF7970A module. The PCB antenna design is shown in Figure 2. Critical parameters such as trace width, spacing, and impedance matching were carefully considered to minimize signal loss and maximize read range. SPI communication was established between the TRF7970A and the microcontroller, and an interrupt request (IRQ) line was configured to trigger card detection events.

Figure 1. Multi-turn coil antenna (125kHz)

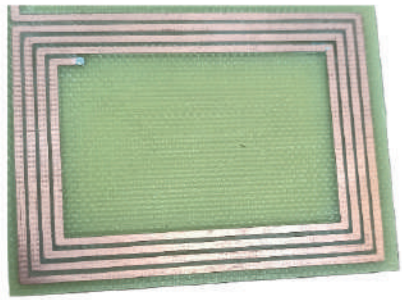

Figure 2. PCB antenna design (13.56Mhz)

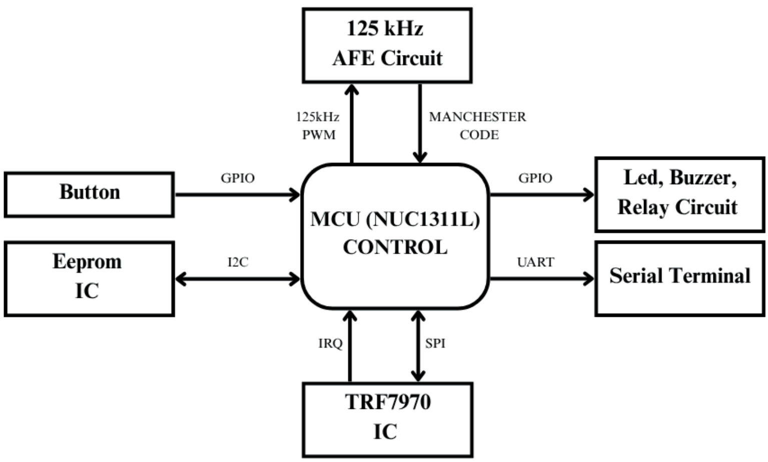

Figure 3. General Block Diagram of the Hardware

In addition to the main RFID modules, the system is equipped with light-emitting diode (LED) indicators, a buzzer circuit, and a relay module for access control feedback. Card registration and deletion are performed via a push-button interface, and card identification numbers (ID) are stored in an external electrically erasable programmable read-only memory (EEPROM) chip. The system compares each scanned card with the EEPROM database to verify authorization status. All relevant information, including the scanned card ID and authorization status, is transmitted via UART to a connected terminal for real-time monitoring. Figure 3 shows the overall block diagram of the design.

Software Components

RFID cards operating at 125 kHz typically use data encoding methods such as Manchester or Non-Return-to-Zero (NRZ). Manchester decoding is used in this study due to its reliability in detecting bit transitions. The microcontroller monitors the rising and falling edges of the input signal to determine each bit: a rising edge is interpreted as a logical '1', a falling edge as a logical '0'. This method allows the transmitted data stream to be accurately reconstructed. After the 64-bit Manchester encoded data stream is decoded, cyclic redundancy check (CRC) verification is performed to ensure data integrity. After successful verification, the 64-bit stream is processed to extract a 32-bit unique identifier (UID) used for access control logic.

The TRF7970A module for high-frequency cards manages communication using ISO/IEC compliant protocols. When a card enters the detection range, the module generates an IRQ, which triggers the microcontroller to initiate SPI-based data reception. The incoming data stream is parsed according to the card's protocol structure and the UID is extracted. After verification and format conversion, the UID is reduced to a 32-bit card ID for uniformity in access processing.

In both frequency domains, the system ensures that only valid and correctly formatted card IDs advance to the access control evaluation phase. To support dual-frequency operation, the system alternates between LF and HF scan cycles. The microcontroller initiates a 200 ms LF scan window and actively checks 125 kHz cards. If no cards are detected within this window, the LF scanner is disabled and the HF scanner is enabled for the next 200 ms. This alternating cycle continues indefinitely. If a card is detected during any of the scan stages, the system temporarily stops scanning to process the card data. Once processing is complete, the alternating scan routine continues from the next cycle.

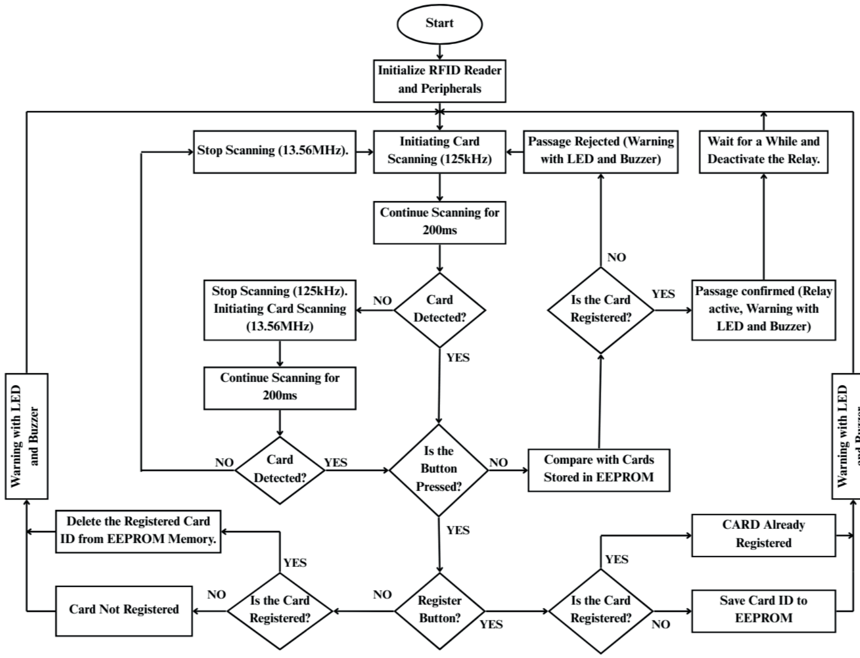

Figure 4. Algorithm Flowchart

The access logic of the system includes card registration, deletion and authorization control. A user can press a physical button to register a new card or delete an existing card. When a card is presented in this mode, the system checks its existence in the EEPROM database. If the card is not registered, it is added to the database. If it is already registered, a "duplicate" warning is given. If deletion is requested, the card is removed from the EEPROM or a "not found" message is returned if it is not stored at all. In normal operation, each card scanned is compared with the list stored in the EEPROM. If the card is recognized, the system authorizes access by triggering the relay, turning on the green LED and briefly sounding the beeper. For unrecognized cards, the system activates the red LED and emits a long beeper tone. All card events and system responses are recorded and transmitted to a serial terminal via UART for monitoring and debugging purposes. Figure 4 shows the general flowchart for the software algorithm.

RESULTS

In this study, a dual-frequency RFID card reader system capable of operating at both 125 kHz and 13.56 MHz was successfully designed, implemented, and tested. All tests and experimental analyzes were carried out at BUTKON R&D center (BUTKON, 2025). The system demonstrated reliable performance in reading EM4100-compliant LF cards, including MIFARE Classic, MIFARE Ultralight, and NFC-enabled cards, as well as ISO/IEC 14443-compliant HF cards. Combining both frequency bands under the control of a single microcontroller enabled seamless protocol transition and real-time access control functionality. Figure 5 shows the designed circuit board, and Figure 6 shows the card ID information read as a result of the tests.

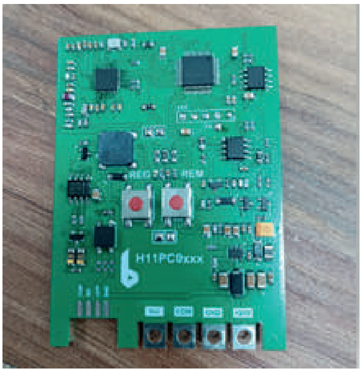

Figure 5. The prototype of the designed circuit

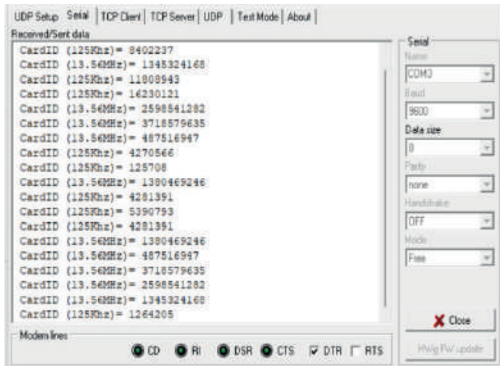

Figure 6. Serial Terminal Output

Considerable emphasis was placed on robust hardware design practices, including careful PCB layout with ground plane separation and controlled return paths to reduce electromagnetic interference. LC resonator and planar PCB antenna structures were optimized to target resonant frequencies and validated through experimental testing using LCR meters and functional range measurements. Terminated antenna designs provided stable performance and adequate read distances in both frequency domains.

Overall, the developed system provides a compact, scalable and cost-effective solution for modern access control applications. Its ability to support multiple card types, conduct secure authentication and withstand harsh environmental conditions makes it suitable for use in factories, smart building entry points and high-traffic areas. The dual-frequency design ensures long-term compatibility with a wide range of existing RFID infrastructures and simplifies system management by combining different card technologies on a unified reader platform.

CONCLUSION AND DISCUSSION

In this study, a multi-frequency and multi-protocol card reader prototype that can read both low-frequency (LF, 125 kHz) and high-frequency (HF, 13.56 MHz) RFID cards was developed. The developed system successfully identified EM4100-based LF cards and HF/NFC compatible cards such as MIFARE Classic and Ultralight; and automatically switched between frequencies. The tests showed that the cards could be read correctly and that the system could operate stably even in metal enclosures against electromagnetic interference. In this way, it offers a practical, flexible and reliable solution for security-oriented applications such as device access control.

While the current system successfully achieves multi-frequency RFID card detection and access control functionality, future studies could focus on extending its capabilities through integration with IoT platforms. Adding IoT support would enable real-time remote monitoring, centralized data logging, and improved traceability of access events. Secure communication protocols can be implemented to transmit card activity to cloud-based databases, allowing for advanced data analytics and audit capabilities. In addition, the development of a user-friendly web or mobile application interface could simplify user registration, card management, and remote authorization processes. Such improvements would enhance the system's scalability and usability in complex or distributed environments, such as smart buildings, campuses, or industrial facilities. Incorporating features like biometric verification or multi-factor authentication in future iterations may also further strengthen system security and versatility.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Mustafa Talip Koyuncu: Methodology, investigation, visualization, writing.

Burak Demir: Methodology, investigation

Muhammet Fatih Aslan: Writing, supervision.

Akif Durdu: Supervision.

Chapter 11

Structural and Statistical Analysis of Finite Mixture Models Based on q-Calculus

Nurgül Okur[1]

Abstract

The foundations of q-analysis date back to the 1740s, when Euler introduced the theory of partitions, also referred to as additive analytic number theory. Over the years, the discovery of q-calculus applications in fields such as operator theory, combinatory, probability theory, and many others has sparked tremendous interest in this mathematical framework.

Mixture distributions are probabilistic models in which a data set is assumed to originate from multiple underlying distributions, each contributing with a certain probability. These distributions are commonly used to model complex data structures more accurately.

This paper introduces q-finite mixture models as a novel extension of the classical finite mixture family, motivated by recent progress in q-calculus and generalized probability distributions. By incorporating a deformation parameter q, the proposed mixture models offer enhanced modeling flexibility for a variety of stochastic phenomena. The fundamental distributional and statistical properties of the suggested q-mixture models are systematically are explored.

INTRODUCTION

Quantum calculus, also known as q-calculus or calculus without limit is a generalization of classical calculus that originated in the early 20th century, although its roots can be traced even further back. Euler studied the q-analog of Newton's infinite series and made foundational contributions. Jacobi formulated the Gauß-Jacobi triple product identity. Gauß introduced the q-binomial coefficients and established identities involving them. Jackson defined the concept of the q-integral. Ernst provided a comprehensive

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historical overview and proposed a new approach to q-calculus. Cheung and Kac authored the monograph Quantum Calculus, further developing the field.

Mixture distributions are probabilistic models in which observations are assumed to originate from multiple underlying distributions, each with a certain probability. The evolution of q-distributions represents a natural progression in the development of q-calculus. q-calculus serves as a parametric generalization of classical calculus, with the classical framework being recovered in the limit as $q \to 1$. Significant contributions to the theory of q-distributions including such as Dunkl, 1981, Crippa et al., 1997, Kupershmidt, 2000, Kemp, 2002, Charalambides, 2016, including the Gaussian and generalized gamma q-distributions by Diaz et al., 2009, 2010, the Erlang q-distributions by Charalambides, 2016, the gamma and beta q-distributions by Boutouria et al., 2018, the Lindley q-distribution in two forms was introduced by Bouzida, 2023.

In response to recent progress in the study of generalized probability q-distributions, this paper presents q-finite mixture distribution with their fundamental statistical and distributional characteristics.

MATERIAL AND METHODS

This section outlines the principles of q-calculus, and q-probability theory. In this entire study, unless otherwise stated, it is assumed that $0 < q < 1$. Readers are referred to the relevant literature.

Definition 1 (Kac and Cheung, 2002). Let x, q be real numbers. The q-number $[x]_q$ is defined as

$$[x]_q = \frac{1-q^x}{1-q}.$$

Definition 2 (Kac and Cheung, 2002). The q-Gauss binomial formula is given by

$$(x+y)_q^n = \sum_{k=0}^{n} \begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}} y^k x^{n-k}, \quad -\infty < x, y < \infty.$$

The q-binomial coefficients are provided for $k = 0, 1, \ldots, n$ by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q = \frac{[n]_q!}{[n-k]_q![k]_q!}, \quad [n]_{k,q} = \frac{[n]_q!}{[n-k]_q!}, \quad [n]_q! = [n]_q [n-1]_q \ldots [2]_q [1]_q.$$

Definition 3 (Kac and Cheung, 2002). The q-analogues of the exponential function are presented

$$E_q^{\tau} = \sum_{k=0}^{\infty} q^{\binom{k}{2}} \frac{\tau^k}{[k]_q!} = \prod_{k=0}^{\infty} \left(1+(1-q)q^k\tau\right), \quad \tau \in \mathbb{R},$$

$$e_q^{\tau} = \sum_{k=0}^{\infty} \tau = \prod_{k=0}^{\infty} \frac{1}{\left(1-(1-q)q^k\tau\right)}, \quad |\tau| < \frac{1}{1-q}.$$

Definition 4 (Kac and Cheung, 2002). The q-derivative of f is defined as

$$D_q f(\tau) = \frac{f(q\tau)-f(\tau)}{q\tau-\tau}$$

Definition 5 (Kac and Cheung, 2002). The well-known Jackson q-integral of f is given by

$$\int_0^b f(\tau)\,d_q\tau = (1-q)\sum_{n=0}^{\infty} q^n b f(q^n b), \; b>0.$$

Definition 6 (Vamvakari, 2023). X is considered q-continuous if there exists $f_q^X(x)$ such that

$$P\{a < X \le b\} = \int_a^b f_q^X(x)\,d_q x, \; x \ge 0.$$

The q-cumulative distribution function (q-CDF) of X is defined for $x>0$

$$F_q^X(x) = P(X \le x) = \int_0^x f_q^X(u)\,d_q u,$$

satisfying the relation $P(\alpha < X \le \beta) = F_q^X(\beta) - F_q^X(\alpha)$. Then, $f_q(x) = D_q F_q(x)$.

Definition 7 (Vamvakari, 2023). Under the condition $\xi_{(1)} < q\xi_{2)} < \xi_{(2)} < \ldots < \xi_{(n-1)} < q\xi_{(n)}$ the random variable $\xi_{(\nu)}$ is defined as ν-th q-ordered random variable. The q-CDFs of the q-ordered statistics $\xi_{(n)}$, $\xi_{(1)}$, $\xi_{(\nu)}$(1 ≤ $\nu \le n$) are expressed, respectively

$$F_q^{\xi_{(n)}}(\tau)=\prod_{i=1}^{n}F_q\left(q^{i-1}\tau\right),\ \ F_q^{\xi_{(1)}}(\tau)=1-\prod_{i=1}^{n}\left(1-F_q(\tau)\right),$$

$$F_q^{\xi_{(v)}}(\tau)=\sum_{w=v}^{n}\sum_{1<i_1<\ldots<i_w<n}\prod_{j=1}^{r}F_q\left(q^{j-1}\tau\right)\prod_{m=w+1}^{n}\left(1-F_q\left(q^{i_m-(m-w)}\tau\right)\right)$$

Definition 8 (Vamvakari, 2023). Let $\mathbb{E}_q\left|\xi^r\right|<\infty$ for all positive integers r. Then,

$$\mu_q^{(r)}=\mathbb{E}_q\left(\xi^r\right)=\int_0^\infty \tau^r f_q^{\xi}(\tau)\,d_q\tau,\ \ \mathbb{E}_q(\xi)=\mu_q,\ \ \mathbb{V}_q(\xi)=\mu_q^{(2)}-\left(\mu_q\right)^2.$$

Definition 9 (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative RV, and $\mathbb{E}_q\left|\left(\xi-\mu_q\right)_q^r\right|<\infty$ for all positive integers r. Then,

$$m_q^{(r)}=\mathbb{E}_q\left(\xi-\mu_q\right)_q^r=\sum_{s=0}^{r}\begin{bmatrix} r\\ s\end{bmatrix}_q q^{\binom{s}{2}}(-1)^s\,\mu_q^s\mu_q^{(r-s)},\ \ s\le r.$$

Definition 10 (Okur and Djongmon, 2025). Let ξ be a q-continuous non-negative RV. Then, its q-MGF is expressed in two distinct forms as follows:

$$\mathbb{M}_q^{I}(t)=\mathbb{E}_q\left(E_q^{qt\xi}\right)=\int_0^\infty E_q^{qt\tau}f_q(\tau)\,d_q\tau,\ \mathbb{M}_q^{II}(t)=\mathbb{E}_q\left(e_q^{t\xi}\right)=\int_0^\infty e_q^{t\tau}f_q(\tau)\,d_q\tau.$$

Definition 11 (Djongmon and Okur, 2025). Let ξ be a q-continuous non-negative RV. Then,

the q-survival function (q-CCDF) $\mathbb{S}_q(\tau)=P(\xi>\tau)=1-F_q(\tau),$

the q-hazard rate function (q-HRF)

$$h_q(\tau)=\frac{P\left(q\tau\le\xi\le\tau\,\middle|\,\xi\ge q\tau\right)}{(1-q)\tau}=\frac{f_q(\tau)}{\mathbb{S}_q(q\tau)},$$

the q-mean residual life function (q-MRLF)

$$mrl_q(\tau)=\mathbb{E}_q\left(\xi-\tau\,\middle|\,\xi>\tau\right)=\frac{1}{\mathbb{S}_q(\tau)}\int_\tau^\infty \mathbb{S}_q(qu)\,d_qu.$$

Definition 12 (Djongmon and Okur, 2025). Let X and Y be two independent q-continuous non-negative random variables. The q-stress-strenght reliability (q-SSR) is given by:

$$\mathbb{R}_q = P\left(X > Y\right) = \int_0^\infty P\left(X > \tau \middle| Y = \tau\right) f_q^X\left(\tau\right) d_q\tau = \int_0^\infty f_q^X\left(\tau\right) \mathrm{F}_q^Y\left(\tau\right) d_q\tau.$$

RESULTS

This section outlines a q-finite mixture model, including structral and statistical properties.

Modeling of a q-Finite Mixture Model

A q-finite mixture model (denoted q-FMM) is a q-probabilistic model composed of multiple component q-distributions combined with certain weights. The general form of mixture q-PDF is described as:

$$f_q^{mix(i_m)}\left(x\right) \quad = \sum_{k=1}^{K} \pi_{q_i}^{(k)} \cdot f_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right),\ q_i \in \left\{q, 1/q\right\},\ 0 < q < 1$$

where:

$i \in \left\{I, II\right\}$: type of q-mixture model such that q, for $i = I$ and $1/q$, for $i = II$

$m \in \left\{1, 2, \cdots, 2^n\right\}$: number of q-mixture model

K: number of components,

$\pi_{q_i}^{(k)}$: mixing proportion of the k-th component, $\pi_{q_i}^{(k)} \geq 0$, $\sum_{k=1}^{n} \pi_{q_i}^{(k)} = 1$

C_m: selecting function the appropriate q or $1/q$-PDF for the k-th density component

$\lambda_{(k)}$: parameter vector of the k-th density component

$f_q^{(k)}\left(x;\lambda^{(k)}\right)$: the k-th q-density component

$f_{1/q}^{(k)}\left(x;\lambda^{(k)}\right)$: the k-th $1/q$-density component

As $q \to 1$, the q-finite mixture model converges to its ordinary form.

Structure Properties of the q-Finite Mixture Model

The q-finite mixture model represents a general modeling framework that subsumes both q-homogeneous and q-hybrid variants, offering enhanced flexibility for representing heterogeneous systems characterized by different q-parametrizations. Let $\mathbb{I}_A(k)$ be the indicator function equal to 1 if $k \in A$, and 0 otherwise, and $\mathbb{I}_B(k)$ is similarly defined for complementary set $B \subset \{1,2,\dots,K\}$, where $A \cup B = \{1,2,\dots,K\}$.

q-Homogeneous Finite Mixture Model. A q-homogeneous mixture model refers to a class of mixture models in which all component distributions are derived exclusively from a single formulation family—either the original q-formulation or its reciprocal counterpart based on $1/q$. The homogeneity of the component structure ensures a consistent parametric behavior across the mixture, which is particularly advantageous for analytical tractability and interpretability within the same functional family. Hence, it can be formulated:

$$f_q^{mix(i_m)}(x) = \sum_{k=1}^{K}\left(\pi_q^{(k)} \cdot \mathbb{I}_A(k) + \pi_{1/\mathrm{q}}^{(k)} \cdot \mathbb{I}_B(k)\right) \cdot f_{q_i}^{(k)}\left(x;\lambda^{(k)}\right).$$

q-Hybrid Finite Mixture Model. A q-hybrid mixture model is a type of mixture model that incorporates component distributions originating from both the original q-formulation and its reciprocal counterpart based on $1/q$. Unlike the q-homogeneous model, the q-hybrid structure allows for the coexistence of multiple generative mechanisms within a single framework. Thus, it can be defined as:

$$f_q^{mix(i_m)}(x) = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot \left(f_q^{(k)}\left(x;\lambda^{(k)}\right) \cdot \mathbb{I}_A(k) + f_{1/q}^{(k)}\left(x;\lambda^{(k)}\right) \cdot \mathbb{I}_B(k)\right).$$

Statistical Properties of the q-Finite Mixture Model

The corresponding q-mixture statistical characteristics are given by

q-cumulative distributional function of model and the q-ordered statistics

$$F_q^{mix(i_m)}(x) = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot F_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right), F_q^{\xi_{(1)}(mix(i_m))}(x) = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot F_{q_i}^{\xi_{(1)}(C_m(k))}\left(x;\lambda^{(k)}\right)$$

$$F_q^{\xi_{(n)}(mix(i_m))}(x) = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot F_{q_i}^{\xi_{(n)}(C_m(k))}\left(x;\lambda^{(k)}\right), F_q^{\xi_{(\nu)}(mix(i_m))}(x) = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot F_{q_i}^{\xi_{(\nu)}(C_m(k))}\left(x;\lambda^{(k)}\right)$$

q-moment, q-central moment, q-expectation and q-variance:

$$\mu_q^{(r)(mix(i_m))} = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot \mu_{q_i}^{(r)(C_m(k))}, \; m_q^{(r)(mix(i_m))} = \sum_{k=1}^{K}\pi_{q_i}^{(k)} \cdot m_{q_i}^{(r)(C_m(k))}$$

$$\mathbb{E}_q^{mix(i_m)}(\xi)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{E}_{q_i}^{C_m(k)}\left(\xi^{(k)}\right),\ \mathbb{V}_q^{mix(i_m)}(\xi)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{V}_{q_i}^{C_m(k)}\left(\xi^{(k)}\right)$$

q-moment generating function:

$$\mathbb{M}_q^{I(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{M}_{q_i}^{I(C_m(k))}(t),\ \mathbb{M}_q^{II(mix(i_m))}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{M}_{q_i}^{II(C_m(k))}(t)$$

q-reliability functions and q-stress-strength reliability:

$$\mathbb{S}_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{S}_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right),\ h_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot h_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right)$$

$$mrl_q^{mix(i_m)}(x)=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot mrl_{q_i}^{C_m(k)}\left(x;\lambda^{(k)}\right),\ \mathbb{R}_q^{mix(i_m)}=\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathbb{R}_{q_i}^{C_m(k)}$$

An Illustirative Example (q-Exponential Finite Mixture Model)

A q-exponential finite mixture model (q-EFMM) is constructed by combining multiple q-exponential distributional components through a weighted linear combination. These weights reflect the relative contributions of each component and may be generalized using the q-algebra framework. The resulting model provides a flexible representation for heavy-tailed data. Its general form is:

$$\xi\sim\sum_{k=1}^{K}\pi_{q_i}^{(k)}\cdot\mathrm{Exp}_q\left(x;\lambda^{(k)}\right),\ q_i\in\{q,1/q\},\ 0<q<1$$

where $\mathrm{Exp}_q\left(x;\lambda^{(k)}\right)$ represents th exponential q-distribution and the corresponding q and $1/q$-component densities are:

$$f_q^{(k)}\left(x;\lambda^{(k)}\right)=\lambda^{(k)}E_q^{-q\lambda^{(k)}x}\cdot\mathbb{I}_{[0,1/1-q]}(x),$$

$$f_{1/q}^{(k)}\left(x;\lambda^{(k)}\right)=\lambda^{(k)}e_q^{-\lambda^{(k)}x}\cdot\mathbb{I}_{[0,\infty)}(x).$$

For $K=2$, let us describe two types of the q-mixture exponential model with a fixed mixing proportion. For the sake of simplification, let

$$\pi_q^{(1)}=\omega_q,\pi_q^{(2)}=1-\omega_q\ ,\ \pi_{1/q}^{(1)}=\omega_{1/q},\pi_{1/q}^{(2)}=1-\omega_{1/q}$$

Thus, the parameters of the q-density components may either be identical or non-identical. In such a case, $\lambda^{(1)}=\lambda^{(2)}=\lambda>0$ or alternatively, $\lambda^{(1)}=\alpha\neq\lambda^{(2)}=\beta$, where $\alpha,\beta>0$. Accordingly, the distributional characteristics of the q-exponential mixture model (q-EMM) are given by:

Table 1. The exponential mixture q-PDFs with identical component parameters

m	The Homogeneous Mixture q-PDF	The Hybrid Mixture q-PDF
1	$\lambda E_q^{-q\lambda x}$	$\omega_q\lambda E_q^{-q\lambda x}+\left(1-\omega_q\right)\lambda e_q^{-\lambda x}$
2	$\lambda e_q^{-\lambda x}$	$\omega_q\lambda e_q^{-\lambda x}+\left(1-\omega_q\right)\lambda E_q^{-q\lambda x}$
3	$\lambda E_q^{-q\lambda x}$	$\omega_{1/q}\lambda E_q^{-q\lambda x}+\left(1-\omega_{1/q}\right)\lambda e_q^{-\lambda x}$
4	$\lambda e_q^{-\lambda x}$	$\omega_{1/q}\lambda e_q^{-\lambda x}+\left(1-\omega_{1/q}\right)\lambda E_q^{-q\lambda x}$

Table 2. The exponential mixture q-PDFs with non-identical component parameters

m	The Homogeneous Mixture q-PDF	The Hybrid Mixture q-PDF
1	$\omega_q\alpha E_q^{-q\alpha x}+\left(1-\omega_q\right)\beta E_q^{-q\beta x}$	$\omega_q\alpha E_q^{-q\alpha x}+\left(1-\omega_q\right)\beta e_q^{-\beta x}$
2	$\omega_q\alpha e_q^{-\alpha x}+\left(1-\omega_q\right)\beta e_q^{-\beta x}$	$\omega_q\alpha e_q^{-\alpha x}+\left(1-\omega_q\right)\beta E_q^{-q\beta x}$
3	$\omega_{1/q}\alpha E_q^{-q\alpha x}+\left(1-\omega_{1/q}\right)\beta E_q^{-q\beta x}$	$\omega_{1/q}\alpha E_q^{-q\alpha x}+\left(1-\omega_{1/q}\right)\beta e_q^{-\beta x}$
4	$\omega_{1/q}\alpha e_q^{-\alpha x}+\left(1-\omega_{1/q}\right)\beta e_q^{-\beta x}$	$\omega_{1/q}\alpha e_q^{-\alpha x}+\left(1-\omega_{1/q}\right)\beta E_q^{-q\beta x}$

The graphs below illustrate the q-exponential mixture model with identical component parameters for $\omega=0.75$ and $\lambda=2$, with varying values of the q-parameter as outlined in Table 1.

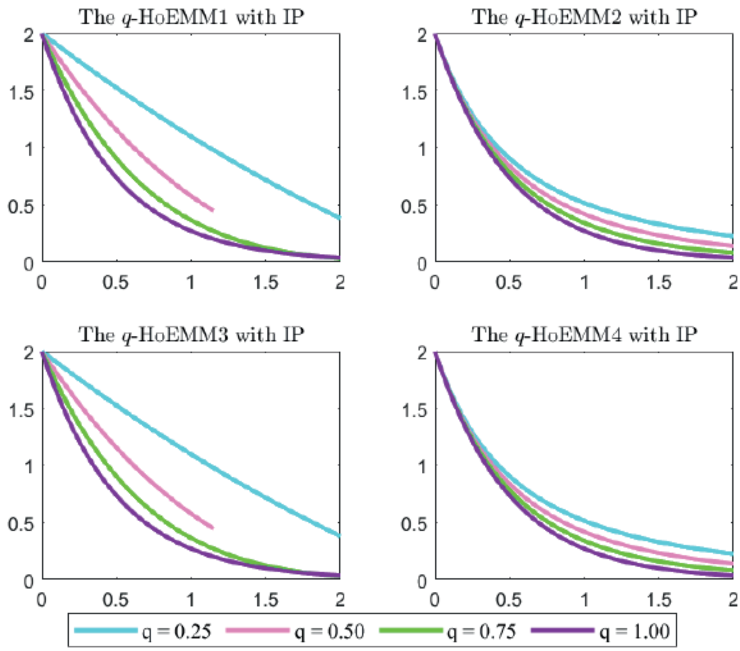

Figure 1. Graph of the homogeneous exponential mixture q-PDFs with identical component parameters for $\omega = 0.75$ and $\lambda = 2$

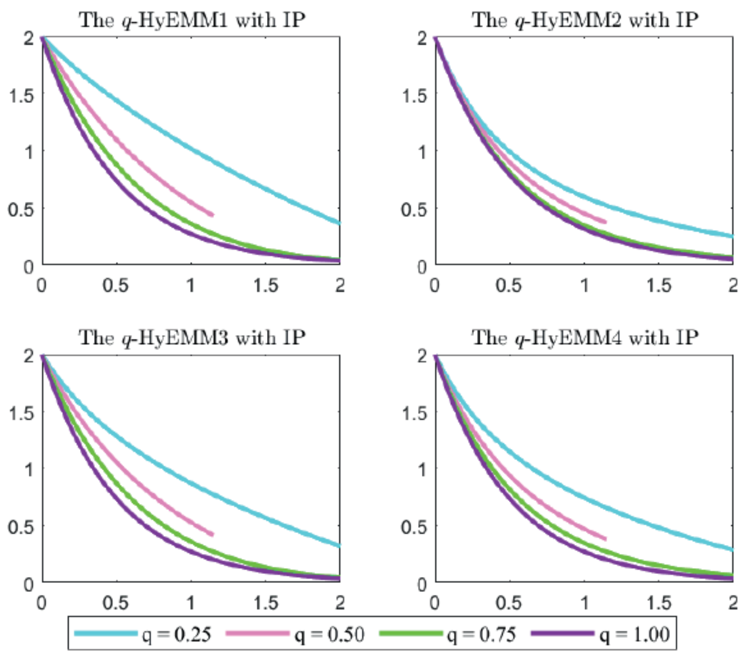

Figure 2. Graph of the hybrid exponential mixture q-PDFs with identical component parameters for $\omega = 0.75$ and $\lambda = 2$

Presented below is the graph of the q-exponential mixture model with non-identical component parameters corresponding to $\omega = 0.75$ and $\alpha = 1, \beta = 3$, under varying q-parameter values as specified in Table 2. The homogeneous and hybrid configurations are depicted in Figures 3 and 4:

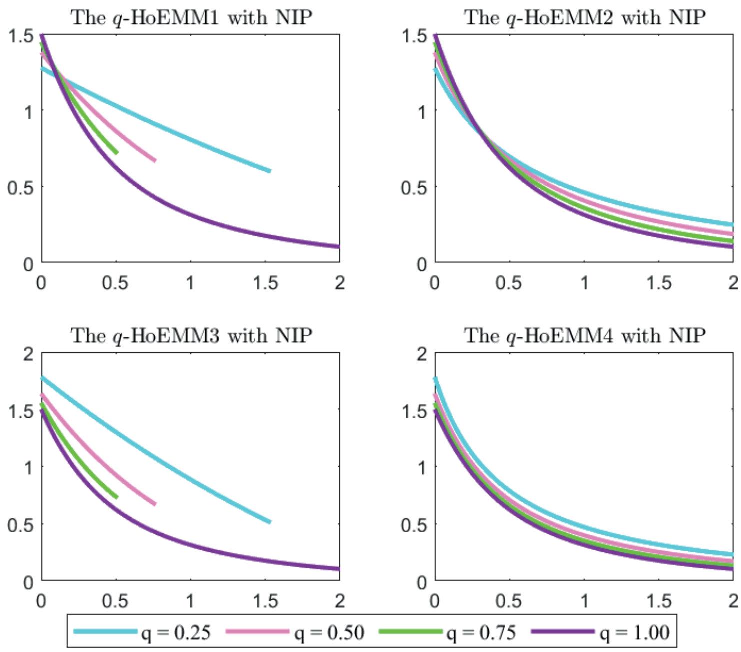

Figure 3. Graph of the homogeneous exponential mixture q-PDFs with non-identical component parameters for $\omega = 0.75$ and $\alpha = 1, \beta = 3$

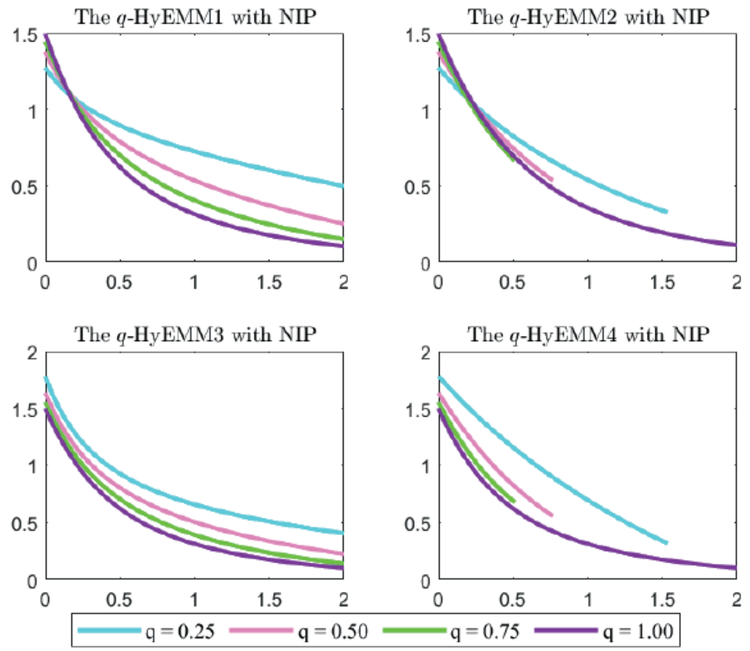

Figure 4. Graph of the hybrid exponential mixture q-PDFs with non-identical component parameters for $\omega = 0.75$ and $\alpha = 1, \beta = 3$

Above, q-HoEMMm $(m = 1-4)$ and q-HyEMMm $(m = 1-4)$ denote homogeneous and hybrid q-exponential mixture models, respectively. NIP and IP stand for non-identical and identical parameters. The plots show that increasing q enhances model convexity, and all q-exponential mixtures reduce to the standard form as $q \to 1$.

DISCUSSION AND CONCLUSION

Probability q-distributions provide a flexible and dynamic framework that generalizes classical probability distributions by introducing the q-parameter. This parameter allows for a broader class of probabilistic models, enriching both theoretical understanding and practical applications.

In this paper, we introduce q-finite mixture model, and and provide a detailed analysis of the structural and statistical properties. As a representative example to elucidate the core concept, this study presents an exponential mixture model with a fixed mixing proportion, along with a discussion of its properties. Our findings suggest that the proposed q-distribution holds significant promise and may have widespread applications across various fields. In future research, we aim to explore the finite mixture and compound q-distribution.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to the finalization of this paper.

Chapter 12

Sustainable Beauty Concept: A Conceptual Review Through Environmental, Social, and Economic Dimensions

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Dursun Yılmaz[2]

Abstract

This study aims to examine the increasingly significant concept of sustainability in the beauty industry within a multidimensional framework encompassing environmental, social, and economic aspects. As a response to the environmental and ethical issues posed by conventional beauty practices, the sustainable beauty approach has emerged, shaped around principles such as the use of natural ingredients, fair supply chains, ethical production conditions, and eco-friendly packaging. The theoretical foundation of the research is built on the concepts of circular economy, green marketing, and ethical consumption. Employing a qualitative research method, this study analyzes the conceptual structure of sustainable beauty based on a comprehensive review of current academic literature and sectoral reports. Findings indicate that demand for sustainable products is particularly increasing among young consumers; however, high costs, lack of information, and trust issues regarding brands limit the widespread adoption of these products. The study emphasizes that for sustainable beauty to be adopted at a societal level, efforts should be made to enhance consumer education, implement transparent and reliable ethical certification systems, promote local production, and develop supportive public policies. In this context, sustainable beauty should be understood as a holistic approach that goes beyond individual preferences, closely tied to lifestyle choices, value systems, and broader societal transformation processes.

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INTRODUCTION

The intensification of the global climate crisis, rapid depletion of natural resources, environmental injustices, and social inequalities have elevated sustainability from merely an environmental concern to a fundamental principle across all spheres of life. While sustainability approaches have long been debated in sectors such as food, fashion, and transportation, recent years have witnessed their growing prominence in the beauty and personal care industry (UNEP, 2021). The heightened sensitivity of Gen Z and Alpha consumers toward eco-friendly and ethically produced goods has been a driving force behind transformative changes across all stages of beauty product development—from formulations and packaging to supply chains and marketing strategies (NielsenIQ, 2023).

Sustainability, in its simplest definition, refers to "meeting the needs of the present without compromising the ability of future generations to meet their own needs" (World Commission on Environment and Development, 1987). Beyond this definition, sustainability is a multidimensional concept encompassing environmental, economic, and social dimensions, necessitating a reevaluation of individual and institutional production-consumption behaviors from both ethical and ecological standpoints. The cosmetics and beauty industry is undergoing a critical transformation in this context. This sector is not limited to individual aesthetic practices but is deeply intertwined with multifaceted challenges, including natural resource exploitation, chemical ingredients, plastic waste generation, and labor exploitation (Akman, 2024; Ellen MacArthur Foundation, 2017).

The rise of terms such as *clean beauty*, *zero waste beauty*, *vegan cosmetics*, and *slow beauty* in industry discourse signals that this shift is not merely a superficial trend but indicative of a profound structural transformation (Sustainable Beauty Coalition, 2022). However, sustainable beauty must extend beyond offering eco-conscious products; it should also encompass transparency in production processes, fair labor practices, ethical marketing strategies, and consumer education (Peattie, 2001).

This study aims to holistically examine the theoretical foundations, practical applications, and critical debates surrounding sustainability in the beauty industry. It elaborates on the environmental, social, and economic dimensions of sustainable beauty, highlighting its distinctions from traditional beauty paradigms. Furthermore, it evaluates industry trends through theoretical lenses such as the circular economy, green marketing, and ethical consumption. The study also critically addresses the risks of reducing sustainable beauty practices to *greenwashing* strategies—superficial marketing tactics that falsely project an eco-friendly image.

The scope of this research is confined to the theoretical underpinnings and exemplary practices of sustainability in the beauty and personal care sector. Primarily grounded in international literature and industry reports, this analysis also briefly touches on local applications. The study does not measure the scientific environmental impacts of production processes but offers a theoretical discourse-level analysis of narratives and strategies. Thus, it provides a conceptual framework for understanding how sustainability is shaping the beauty industry and the ethical, cultural, and structural challenges accompanying this evolution.

CONCEPTUAL FRAMEWORK

The concept of sustainable beauty emerges as a holistic approach that offers an alternative to traditional beauty norms by adopting production-consumption models respectful of both nature and humanity. This approach is grounded in the definition of sustainability presented in the Brundtland Report (1987): "Meeting the needs of the present without compromising the ability of future generations to meet their own needs." Accordingly, sustainable beauty encompasses not only the content of the product but also the entire production cycle, from raw material sourcing to the supply chain, packaging processes, and marketing strategies (World Commission on Environment and Development, 1987).

Environmental Dimension

The environmental impact of the cosmetics industry becomes particularly evident in its production of plastic waste and use of toxic chemicals. According to the Ellen MacArthur Foundation (2017), the sector produces approximately 120 billion units of packaging annually, accounting for nearly 70% of global plastic waste. In contrast, sustainable beauty practices aim to mitigate this environmental burden by promoting the use of recyclable, refillable, or biodegradable packaging. Solid shampoos, soap-based skincare products, and packaging-free cosmetics are among the solutions that align with the "zero waste" principle (L'Oréal, 2022). Furthermore, the preference for eco-friendly, vegan, and organic ingredients over synthetic chemicals in product formulations seeks to protect both user health and ecological balance.

Social Dimension

Sustainable beauty not only centers on nature but also prioritizes human well-being. This dimension is built on ethical production conditions, resistance to labor exploitation, transparent supply chains, and support for local producers. Fair Trade-certified products ensure fair compensation for small-scale producers. For instance, The Body Shop's Community Trade program provides sustainable income for 21,000 producers in 25 countries (The Body Shop, 2021). Moreover, cruelty-free practices—avoiding animal testing—and non-discriminatory employment practices based on gender, age, or ethnicity are critical to social sustainability.

Economic Dimension

The economic dimension of sustainable beauty has become more visible in recent years due to changing consumer behaviors. According to a report by Grand View Research (2023), the sustainable beauty market is expected to reach a value of $22 billion by 2030. This growth is largely driven by younger consumer groups' interest in the "clean beauty" movement. A study by NielsenIQ (2023) indicates that 64% of Generation Z consider environmental and ethical values when making purchasing decisions. This trend compels brands to invest in sustainable product development processes.

Table 1. Comparison of Conventional and Sustainable Beauty Paradigms

Criterion	Conventional Beauty	Sustainable Beauty
Packaging	- Plastic and single-use materials - Waste-generating designs	- Biodegradable or recycled materials - Refillable systems
Formulation	- Synthetic chemicals and petroleum-derived components - Potential toxic effects	- Organic, vegan, and natural ingredients - Eco-friendly and health-safe components
Supply Chain	- Profit-driven, non-transparent production processes - Resource-exploitative models	- Fair trade and traceable supply chains - Social and environmental responsibility-oriented
Testing Processes	- Animal testing-based certification - Prevalence of unethical practices	- Cruelty-free (animal-friendly) testing - Human rights-sensitive and ethical protocols

(Source: Sustainable Beauty Alliance, 2023; Cosmetics Europe, 2021)

This comparative framework reveals that sustainable beauty extends beyond mere aesthetics, embodying a comprehensive ethical, ecological, and economic stance. It reflects a paradigm shift in which environmental consciousness and social responsibility are central to product development and branding. Nevertheless, there is a growing concern that these values may be co-opted for marketing purposes, leading to superficial applications of sustainability. This phenomenon, commonly referred to as *greenwashing*, involves the use of misleading or exaggerated claims to construct an environmentally friendly image, potentially undermining the integrity of the sustainable beauty movement (Peattie, 2001).

LITERATURE REVIEW

The concept of sustainability gained recognition in international literature as a holistic development goal encompassing not only environmental but also economic and social dimensions following the 1987 Brundtland Report. However, its application in the beauty and cosmetics industry represents a relatively recent trend. Since the mid-2000s, European Union environmental regulations, the pursuit of natural ingredients, and growing opposition to animal testing have catalyzed the adoption of sustainable practices in this sector (Peattie, 2001; PETA, 2022). More recently, this shift has evolved beyond production processes to encompass a cultural transformation influencing consumer behavior, marketing strategies, and even aesthetic perceptions.

This transformation has been reflected in Turkish academia as well. Research has particularly focused on sustainable packaging, natural product formulations, and green marketing strategies. For instance, Kılıç (2022) demonstrated in her study on sustainable packaging designs that biodegradable packaging creates positive impressions among young female consumers. Similarly, Özçelik (2020) analyzed the relationship between green marketing strategies and female purchasing behavior, finding that “natural ingredients” and “cruelty-free” labels were prioritized factors.

Sustainability is also a significant issue in the context of marketing and brand image. Zengin and Aksoy (2021) emphasize that sustainable production policies contribute positively to brand image within the framework of sustainable development, particularly in terms of green marketing and green finance. However, the literature also indicates that some brands may use the concept of sustainability in a superficial manner, misleading consumers—a practice referred to as “greenwashing.” Such critical perspectives have been further developed on a theoretical level in studies suggesting that the discourse of sustainability can, over time, transform into a form of power that exerts moral pressure on consumers—appearing ethical on the surface but potentially deepening social inequalities (Akgün & Bütüner, 2024).

Consumer behavior constitutes another key area of analysis. Studies on Gen Z's attitudes toward sustainable cosmetics in Turkey reveal high environmental awareness within this demographic, though premium pricing remains a barrier to access (Yalçın & Güneri, 2022). This issue is not unique to Turkey; globally, the higher cost of sustainable products has sparked criticism, with concerns that sustainability may become synonymous with luxury consumption (González et al., 2019).

At the industry level, several Turkish brands—including Eyüp Sabri Tuncer, Atelier Rebul, Beeo Apiterapi, The Sim Co., and Herbaflora—exemplify sustainable practices through natural ingredients, recycled packaging, and vegan/cruelty-free production. Reports by TÜSİAD (2021) and TOBB (2020) highlight these advancements while underscoring the need for further R&D and incentives. Additionally, TÜBİTAK-funded projects demonstrate how domestic sourcing of natural cosmetic ingredients aligns sustainability with economic development strategies.

In conclusion, literature on sustainability in the beauty industry continues to diversify, redefining the concept across production, marketing, and consumption paradigms. While Turkish academic contributions offer both critical and practical insights, the field clearly demands more rigorous research. Future work must integrate not only environmental but also social justice and cultural transformation dimensions.

THEORETICAL FOUNDATIONS

The concept of sustainable beauty should be approached not merely as an environmental strategy but as a holistic paradigm that transforms consumption habits, corporate responsibility frameworks, and societal values. This transformation can be understood more deeply through various theoretical lenses. This section examines the key theoretical approaches that underpin sustainable beauty: the circular economy model, green marketing theory, ethical consumption theory, and critical ecology theory.

Circular Economy Approach

The circular economy approach is defined as a production-consumption model that challenges the traditional "take-make-dispose" linear system, instead viewing waste as a resource and prioritizing cyclical processes. This framework promotes the reuse, recycling, and regeneration of products to enhance resource efficiency (Geissdoerfer et al., 2017). In the cosmetics sector, applying this model extends beyond reducing packaging waste to minimizing energy and resource consumption.

According to the Ellen MacArthur Foundation (2019), cosmetics and packaging designed with "cradle-to-cradle" principles not only reduce environmental impacts but also pioneer innovative sustainability practices. In Türkiye, TÜBİTAK-supported projects have advanced circular economy goals by sourcing natural cosmetic ingredients from indigenous plants (Ateş & Karahan, 2022). For instance, essential oils derived from lavender, laurel, and sage are being integrated into eco-friendly formulations, bolstering sustainable production, enhancing domestic manufacturing capacity, and reducing import dependency. Thus, the circular economy offers environmental, economic, and strategic benefits.

Green Marketing Theory

Green marketing theory encompasses strategies for promoting products, services, and brands that prioritize environmental sustainability. Peattie (2001) outlines three evolutionary phases: ecological marketing, environmental marketing, and sustainable marketing. Today, sustainable beauty practices align with the final phase, embedding environmental responsibility across all stages—from production to consumption—rather than relying solely on marketing rhetoric.

Studies conducted in Turkey reveal the impact of green marketing strategies on consumer behavior and highlight the challenges in this field. For instance, Tirkeş (2008), in her doctoral dissertation on organic food, demonstrated that consumer tendencies toward eco-friendly products have increased and that environmental sensitivity plays a decisive role in marketing strategies. Bayazıt and Özdemir (2024) emphasized the influence of environmental awareness, perceptions of social responsibility, and lifestyle choices on consumer preferences. Similarly, Kocaşahin (2019), as well as Çuhadar and Kaytancı (2023), noted that although green marketing has not yet been fully institutionalized in Turkey, these strategies are increasingly being adopted due to the growing environmental consciousness. In their study, Aymankuy et al. (2016) found a direct relationship between environmental attitudes and loyalty to green brands, indicating that nature-centered (ecocentric) approaches are more effective than human-centered (anthropocentric) ones. The research conducted by Duru and Şua (2013) also revealed that young consumers are more sensitive to green marketing practices. These findings suggest that the understanding of green marketing in Turkey is developing both theoretically and practically.

Ethical Consumption Theory

Ethical consumption theory posits that individuals should base purchasing decisions not only on personal needs but also on ethical principles such as environmental sustainability, animal rights, and social justice (Shaw et al., 2005). Criteria like cruelty-free testing, fair trade compliance, and recyclable packaging are increasingly influencing consumer choices.

Research in Türkiye corroborates this trend. Altay (2023) found that Generation Z prioritizes ethical considerations in cosmetics, favoring animal-friendly, vegan, and sustainably sourced brands. Demirci (2023) highlighted how Gen Z's ethical and environmental concerns enhance brand loyalty. Globally, over 60% of consumers prefer brands aligned with ethical principles (Niinimäki, 2020). These findings indicate that ethical consumption theory shapes not only individual behavior but also corporate marketing and product development strategies.

Critical Ecology Theory and Consumption Critique

Critical ecology theory warns against the capitalist co-optation of sustainability discourses to serve capital interests. It argues that positioning sustainable beauty products as luxury items through premium pricing risks exacerbating social inequalities. Anantharaman (2018) contends that sustainable consumption practices intertwine with class and cultural capital, reducing sustainability to a market-driven "image strategy" rather than genuine transformation. Similarly, Rose & Cachelin (2018) critiques the commodification of sustainability, which obscures structural issues and impedes environmental justice.

In Türkiye, marketing discourses for eco-friendly products often prioritize consumer environmental awareness over substantive change, fostering consumer skepticism (Kaya & Demirtaş, 2023). These critiques underscore that sustainable beauty transcends aesthetics or environmentalism, demanding a multilayered transformation encompassing ethical, economic, and cultural dimensions.

General Evaluation

These theoretical frameworks reveal the multifaceted nature of sustainable beauty. The concept must be evaluated not only through aesthetic or environmental lenses but also via ethical, economic, and cultural prisms. Sustainable beauty is integral to a holistic transformation process spanning individual consumption choices, corporate strategies, and global environmental policies.

MATERIAL AND METHODS

This study employs a qualitative research design to examine the theoretical and practical dimensions of sustainable beauty through a multidimensional approach. The research was conducted using document analysis methodology, systematically reviewing national and international academic publications, industry reports, official statistics, and documents from non-governmental organizations.

Research Approach

The study adopts document analysis, a qualitative research technique particularly suitable for conceptual clarification, tracking developments in literature, and conducting comparative analyses (Yıldırım & Şimşek, 2021).

Data Collection Process

Data were selected from studies published between 2017-2024 focusing on sustainable beauty, green marketing, circular economy, and ethical consumption. Sources included peer-reviewed articles, reports, and industry documents accessed through academic databases (Google Scholar, ULAKBİM, Scopus, and Web of Science). Additionally, reports from Turkish sustainability platforms (e.g., Sustainability Academy, Green Thought Association) were incorporated.

Sampling and Selection Criteria

Rather than empirical sampling, the study utilizes a purposive sampling approach based on documents with high conceptual relevance. Selected sources directly addressed sustainability principles in connection with the beauty industry, with particular emphasis on recent Turkish research.

Data Analysis

Data were analyzed using descriptive methods. Documents were categorized according to environmental, social, and economic dimensions of sustainability. Recurring themes and emphases were identified to develop an integrated synthesis, while each document's conceptual framework was examined in relation to theoretical foundations.

Validity and Reliability

Source selection prioritized current (2017-2024), peer-reviewed publications and authoritative industry reports. Each source was evaluated based on publication date, institutional origin, and scholarly impact. Methodological rigor was ensured through comparative analysis of diverse sources and adherence to academic citation standards.

RESULTS

In this study, the concept of sustainable beauty has been examined through environmental, social, and economic dimensions. The sector's current practices were analyzed based on global data and corporate examples.

Environmental Sustainability and the Problem of Plastic Waste

The cosmetics industry has a significant share in plastic packaging consumption. According to *The New Plastics Economy* report by the Ellen MacArthur Foundation (2016), 95% of plastic packaging is discarded after a single use, resulting in an economic loss of approximately $80–120 billion annually. Of the 78 million tons of plastic packaging introduced each year, only 14% is recycled, while most of the remainder is incinerated or released into the environment (Ellen MacArthur Foundation, 2016). These findings highlight the need for a fundamental transformation in the cosmetics industry for environmental sustainability.

Corporate Policies and Best Practices

Leading corporations such as L'Oréal have committed to using 100% recycled plastics in packaging as part of their sustainability goals. According to the company's 2022 Sustainability Report, 79% of product packaging has been made refillable, recyclable, or compostable. Additionally, Garnier introduced its Ultra Soft shampoo series in cardboard tubes in Europe, marking a significant step toward reducing plastic consumption (L'Oréal, 2022).

Similarly, The Body Shop's Community Fair Trade (CFT) initiative, launched in 1987, sources raw materials from local producers based on fair trade principles. As of 2020, the program has collaborated with 26 community-based supplier organizations across 30 countries (The Body Shop, 2023).

Consumer Trends and Market Insights

According to NielsenIQ (2023), 62% of global consumers believe that sustainability has become more important than in previous years. This trend is particularly evident among Generation Z, whose purchasing decisions are increasingly influenced by factors such as natural ingredients, cruelty-free testing, and recyclable packaging (NielsenIQ, 2023).

Furthermore, a report by Grand View Research (2023) estimates that the clean beauty market, valued at USD 8.25 billion in 2023, is expected to reach USD 21.29 billion by 2030. This growth reflects a shift in consumer preferences toward sustainable and nature-based beauty products (Grand View Research, 2023).

Comparative Analysis of Sustainability Dimensions

In light of the data above, the distinctive features of sustainable beauty compared to traditional beauty practices are analyzed in three main categories:

Table 2. Comparative Analysis of Conventional and Sustainable Beauty Practices

Criterion	Conventional Beauty	Sustainable Beauty
Environmental Impact	- High plastic consumption in packaging - Persistent ecological degradation from non-biodegradable waste	- Recyclable/compostable packaging solutions - Carbon-neutral production initiatives
Ingredients	- Petroleum-derived compounds and synthetic additives - Potential allergenicity and long-term health risks	- Plant-based, organic, and non-toxic formulations - Biocompatible ingredients with minimal ecological toxicity
Ethical Production	- Standardized animal testing practices (in vitro/animal models) - Exploitative labor practices with limited transparency	- Certified cruelty-free production protocols (Leaping Bunny/PEA) - Fair trade partnerships and equitable labor policies

Sources: Sustainable Beauty Alliance (2023); Ecocert Cosmetics Standards (2022); Peattie (2001).

These findings indicate that the cosmetics industry is undergoing a gradual transformation aligned with sustainability goals. This shift is driven not only by corporate practices but also by evolving consumer behaviors. However, the industry as a whole still requires substantial structural changes to achieve broader environmental and ethical standards.

DISCUSSION

The findings of this study reveal that the concept of sustainable beauty is gaining increasing importance at both individual and societal levels. This rise in importance is closely linked to growing consumer awareness of environmental issues and a heightened interest in ethical production and packaging processes. In particular, the preferences of Generation Z consumers, who prioritize environmentally friendly and cruelty-free products in their purchasing decisions, reflect the third phase of green marketing proposed by Peattie (2001), which emphasizes consumer-oriented sustainability.

The data presented by NielsenIQ (2023) supports this trend, indicating that 62% of global consumers now regard sustainability as more important than in previous years. Our research also echoes this shift, highlighting how sustainability-driven values are reshaping consumer behavior in the beauty industry. However, despite this growing awareness, the transition from perception to practice remains limited, especially in middle-income economies.

One of the most significant structural barriers to the widespread adoption of the sustainable beauty concept is the generally higher cost of sustainable products. Recent studies conducted in the context of Turkey show that consumers from the middle-income group are less likely to prefer products with sustainable ingredients. Deloitte Turkey's 2023 "Sustainable Consumer" report emphasizes that, although sustainable lifestyles are becoming more widespread, consumers' ability to purchase sustainable products depends largely on their affordability and accessibility (Deloitte, 2023). This situation creates a risk of limiting accessibility and reproducing socioeconomic inequalities, particularly when sustainability is integrated into luxury consumption. Indeed, this critical perspective aligns with theoretical discussions suggesting that, although sustainability is presented as an ethically sound choice, its market-driven structure may, in practice, legitimize and even deepen existing inequalities (Akgün & Bütüner, 2024).

Another challenge lies in the inconsistent and often opaque communication of sustainability claims by companies. While some brands have made significant progress toward sustainable practices—such as L'Oréal's commitment to recyclable packaging and The Body Shop's long-standing Community Fair Trade initiatives—others have been criticized for employing "greenwashing" tactics. As the Ellen MacArthur Foundation (2019) and Peattie (2001) warn, when sustainability becomes a mere marketing tool rather than a genuine corporate value, it risks losing its meaning and undermining consumer trust. In our research, several participants expressed concerns over the lack of transparency in product labeling and information, which they interpreted as a failure of ethical communication.

Additionally, although awareness of sustainable beauty is increasing in Turkey, this has not yet translated into widespread behavioral change. This gap suggests that sustainable beauty must be approached not solely as a product-based trend but as a value-based transformation, supported by education, media discourse, and culturally rooted ecological consciousness. As emphasized by the Green Thought Association (Yeşil Düşünce Derneği, 2022), raising consumer awareness should be considered not only a commercial goal but also a dimension of ecological citizenship.

In summary, the effective mainstreaming of sustainable beauty requires addressing three fundamental issues: economic accessibility, transparency in ethical production, and the establishment of a robust information infrastructure that enables informed consumer choices. Supporting local producers, integrating sustainability into public policy, and empowering civil society actors are essential strategies in this process. Ultimately, achieving meaningful change depends not only on transforming products but also on cultivating sustainable values at both individual and collective levels.

CONCLUSIONS AND RECOMMENDATIONS

This study examined how the concept of sustainability is shaped within the beauty industry, its impact on consumer behavior, and the opportunities and challenges encountered specifically in the Turkish context. The research findings indicate that demand for environmentally friendly products is increasing, particularly among younger consumers. However, this demand is not always met with a supply and production system that aligns with the principles of sustainability. The widespread adoption of sustainable beauty products remains limited due to factors such as high costs, lack of information, and the lack of transparency among certain brands.

The data suggest that the sustainable beauty approach requires not only individual preferences but also structural and cultural transformations. From the perspectives of both producers and policymakers, it is essential to develop fair and accessible practices that promote sustainability. Moreover, media organizations, educational institutions, and non-governmental organizations (NGOs) should be regarded as key actors in fostering a culture of sustainable beauty.

Recommendations:

1. **Consumer Education and Awareness Campaigns:** As the findings show, consumers are familiar with the concept of sustainability but often lack a deeper understanding. Therefore, educational programs and media-supported awareness campaigns targeting especially younger individuals should be organized to help consumers make informed choices regarding sustainable products.
2. **Ethical Certification and Transparency:** Participants highlighted a lack of information about product contents and production processes, revealing the need for greater transparency. Brands should provide

clear and accessible information about their supply chains, ingredients, and production methods. Reliable certification systems such as Fair Trade or EWG Verified should also be promoted and made more visible to the public.

3. **Public Policies and Incentives:** When left solely to market dynamics, sustainable production models tend to have limited impact. Government-supported incentive programs should be expanded, particularly to assist small and medium-sized enterprises (SMEs) in developing sustainable products. These can include grants, tax reductions, and support for research and development (R&D).
4. **Ensuring Price Accessibility:** Findings revealed that consumers from middle-income groups tend to avoid sustainable products due to their higher cost. Therefore, economic regulations such as VAT reductions should be considered, and structural support mechanisms should be introduced to lower production costs and ensure broader accessibility.
5. **Support for Academic Research:** The limited number of academic studies on sustainable beauty in Turkey highlights the need for further research. Interdisciplinary field studies should be encouraged through collaborations between universities, NGOs, and government institutions, with a particular focus on consumer behavior, producer practices, and policy processes.
6. **Promotion of Local Production:** Sustainability should also be viewed in terms of its contributions to local economies. The use of locally sourced, natural, and environmentally friendly ingredients should be supported, and traditional knowledge and nature-compatible production methods should be revitalized. This approach not only contributes to cultural sustainability but also helps reduce dependence on imports.

In conclusion, sustainable beauty is not solely about products but represents a holistic approach that encompasses lifestyle, value systems, and a sense of social responsibility. Achieving this transformation requires a multi-layered effort ranging from individual choices to structural changes in production systems, public policies, and media discourse. Ultimately, sustainable beauty should be embraced as an ethical responsibility that protects not only today's consumers but also the rights of future generations to live in a healthier and more equitable world.

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Conflict of Interest

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Chapter 13

Stationarity Structure of Türkiye's Industrial Production Index

Uğur Ayık[1]

Abstract

Industrial production is considered an important variable in monitoring the economic performance of countries. This study examines the stationarity structures of the indexes of sub-branches of industry as well as the total industrial production index of Türkiye and is a preliminary assessment for the stationarity, cointegration and causality analyses planned to be conducted on this subject in the future. While examining the stationarity structure of the indexes of sub-branches of industry and total industrial production, the series were subjected to separate analyses by taking the level values and logarithmic transformation values (1986M01-2025M02). In order to investigate the stationarity structures of the series, the Ng-Perron Unit Root Test, one of the traditional unit root tests, and the Lee-Strazicich (LS) Unit Root Test, one of the tests that take structural breaks into account, were applied and the results were reported. According to the Ng-Perron test, it was observed that the logarithmic values of the series exhibited a more stationary structure compared to the level values. According to the LS test, it was determined that both the level and logarithmic values were all stationary at the level I(0). In short, the results obtained were that the seasonally adjusted logarithmic industrial production series were generally stationary at the level, the series returned to their averages in the face of shocks, that is, the shocks were eliminated.

INTRODUCTION

In short-term economic policy analyses, industrial production stands out as an important variable. Since certain service activities are closely related to industrial activities, the industrial production sector is important in explaining significant fluctuations in the economy. For this reason, economists continue to evaluate industrial production as a leading indicator

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of economic activity (Bruno and Lupi, 2004; Banerjee et al., 2005; Ejaz and Iqbal, 2021). Economists can use the industrial production index in total or on a sectoral basis in analyses conducted to determine production targets, monitor their development, and evaluate the functioning of the plans made (Kmietowicz, 1995). Industrial production index data is calculated to measure the outputs of the mining, manufacturing, electricity and gas services industries (Tito, 2025).

This study is a preliminary assessment in the determination of stationarity structures for studies that will include monthly data of total industrial production index and sub-branches of industry in their analyses in the coming years, specific to Türkiye. In the analyses, the study size was expanded by investigating the stationarity structures of industry sub-branches in addition to the total industrial production index. Although there are studies that conduct stationarity research based on Türkiye's total industrial production index, the absence of a study examining sub-industry branches differentiates this study from the literature. In important studies conducted for Türkiye (Ertuğrul and Soytaş (2013); Yıldırım and Kılıç (2016); Oğuz (2017)), industrial production index data are generally not stationary at the level and have become stationary by taking the differences of the series. In studies conducted for various countries around the world, as well as Türkiye (Candelon and Gil-Alana (2004); Bulligan et al. (2010); Caporale et al. (2023)), evidence has been obtained that the industrial production index series are stationary at the level or in the first difference.

MATERIAL AND METHODS

In the study, using monthly time series covering the period of January 1986-February 2025, the stationarity structures of the production indices of the industrial sub-branches (Mining and quarrying, Manufacturing industry and Electricity, gas, steam and air conditioning production and distribution) as well as the total industrial production index of Türkiye were examined. In order to determine the stationarity structures of the industrial series, the Ng-Perron test, which is one of the traditional unit root tests, and the LS test, which is one of the unit root tests that take into account structural breaks, were applied. The study data were obtained from the Turkish Statistical Institute (TSI) database and are seasonally and calendar-adjusted series (2021=100 reference year). In addition to the stationarity structure of the level values of the series, the stationarity structures of their logarithmic values were also examined. The analyzes were performed in EViews 12 and RATs software programs.

EMPIRICAL FINDINGS

Before moving on to the stationarity analysis of the industrial production index series, the diagnostic statistical results of the original values of the series are presented and evaluated in Table 1.

Table 1. Diagnostic Statistical Test Results of Series

Variable	Aver.	Med.	Max.	Min.	Std. Dev.	Skewness	Kurtosis	J-B	Prob.
TIPIt	50.60	42.80	112.50	16.50	27.58	0.70	2.22	50.76	0.00
MIN_t	63.15	58.65	104.50	34.70	18.59	0.42	2.01	33.01	0.00
MI_t	50.04	41.80	114.00	16.90	27.74	0.77	2.33	55.64	0.00
EGS_t	53.77	49.15	110.50	11.40	29.19	0.22	1.71	36.37	0.00

Abbreviations: TIPI (total industrial production index), MIN (mining and quarrying), MI (manufacturing industry), EGS (electricity, gas, steam and air conditioning production and distribution), J-B (Jarque-Bera).

The fact that the skewness coefficients of the series presented in Table 1 are greater than zero means that their distribution is skewed to the right, and the fact that the kurtosis coefficients are less than 3 means that their distribution is flatter than normal. When the probability values of the J-B test statistic are examined, it is observed that the basic hypothesis stating that the series have a normal distribution is rejected, and in this case, it is decided that they are not normally distributed. The fact that the series do not have a normal distribution indicates that they are not parametric. Since performing the analysis with nonparametric series may lead to unreliable results, the data must be transformed. At this stage, first of all, the logarithmic transformation method was applied so that the distributions of the series could become normal or close to normal. When the diagnostic statistical tests of the series whose natural logarithm (*ln*) was taken were performed again, it was observed that their skewness ($LnTIPI_t$ skewness = 0.15, $LnMIN_t$ skewness = 0.05, $LnMI_t$ skewness = 0.23, $LnEGS_t$ skewness = -0.12) (as the skewness coefficient approaches zero, it means that the series has a more normal distribution) decreased, that is, it was determined that the series had a distribution close to normal. The graphs of both the level and logarithmic values of the series are presented in Figure 1 and Figure 2, and a preliminary assessment of their stationarity structures was made.

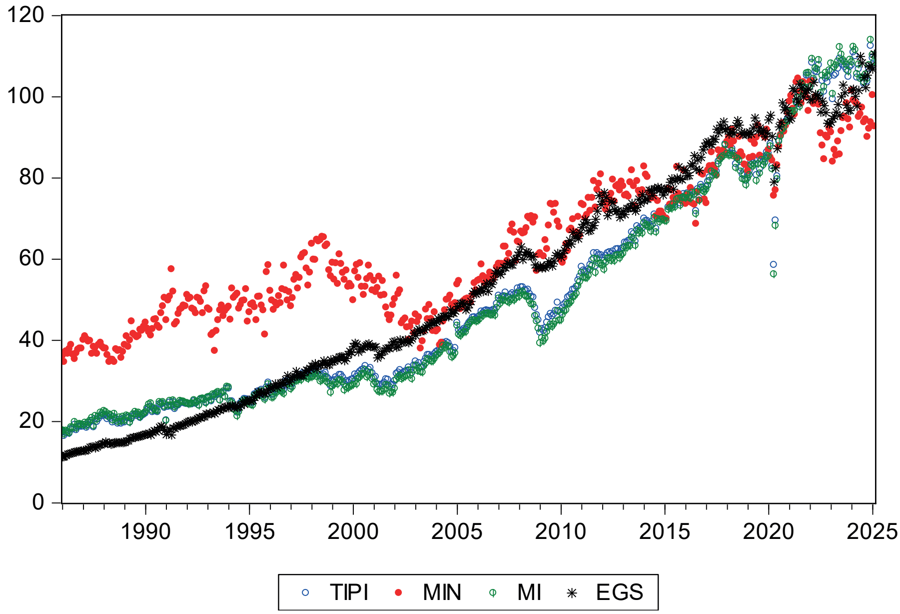

Figure 1. Time Series Graph of Level Values

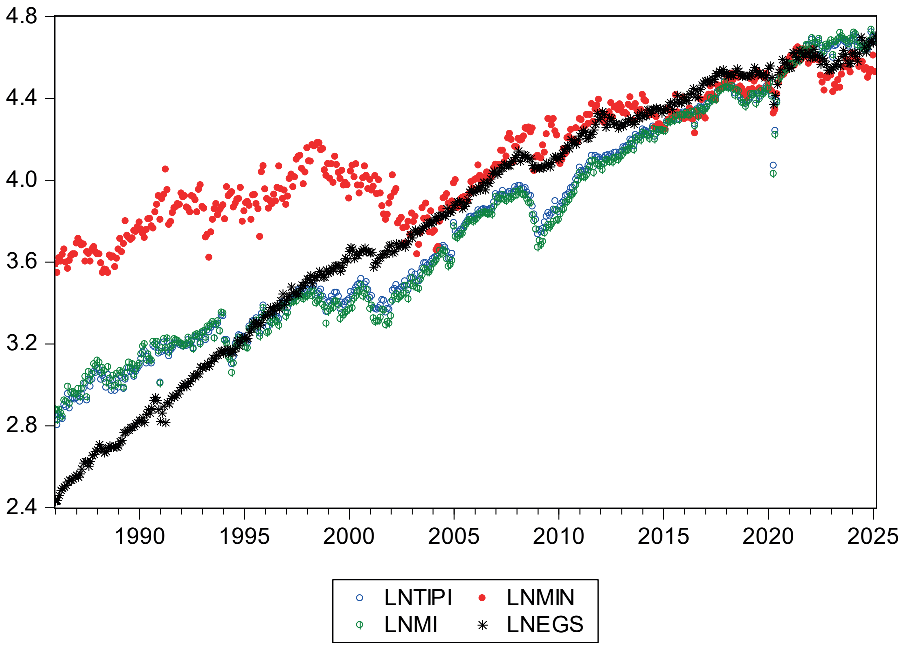

Figure 2. Time Series Graph of Logarithmic Values

When the graphs of the industrial production index time series are examined (Figure 1 and Figure 2), it is suggested that the level and logarithmic values of the $TIPI_t$, MIN_t and MI_t variables generally exhibit an image around their averages, and therefore, a preliminary idea has emerged that they may be stationary at the level. However, it is observed that the level and logarithmic time series of the EGS_t variable exhibit deviations from their averages, that is, it is thought to exhibit a more non-stationary image. In order to express with certainty whether the series contain a unit root, unit root tests were applied to both the level values and the logarithmic transformation values. The obtained stationarity results are presented and evaluated in Table 2.

Table 2. Unit Root Test Results for Industrial Production Index Series

Results for Level Values						
	Ng-Perron (Intercept and Trend)				LS (Model C)	
Variable	MZ_a	MZ_t	*MSB*	*MPT*	t-stat. (TB)	5% Critical Value
$TIPI_t$	-3.53	-1.18	0.33	23.34	-7.51* (2004:11-2008:11)	-5.22
MIN_t	-33.70*	-4.10*	0.12*	2.70*	-8.88* (1997:12-2004:5)	-5.31
MI_t	-3.37	-1.14	0.34	24.17	-7.46* (2004:11-2008:11)	-5.22
EGS_t	-5.63	-1.51	0.27	15.86	-7.37* (2000:9-2017:10)	-5.41
Results for Logarithmic Values						
	Ng-Perron (Intercept and Trend)				LS (Model C)	
Variable	MZ_a	MZ_t	*MSB*	*MPT*	t-stat. (TB)	5% Critical Value
$LnTIPI_t$	-36.43*	-4.26*	0.12*	2.51*	-7.13* (2000:12-2005:2)	-5.22
$LnMIN_t$	-23.66*	-3.43*	0.14*	3.89*	-8.84* (1997:12-2004:5)	-5.31
$LnMI_t$	-29.12*	-3.81*	0.13*	3.13*	-7.08* (2001:2-2008:9)	-5.41
$LnEGS_t$	-0.41	-0.24	0.57	71.20	-7.12* (2000:10-2012:2)	-5.37

Note: Ng-Perron critical values for MZ_a, MZ_t, MSB and MPT tests at 0.05 significance level are -17.300, -2.910, 0.168 and 5.480, respectively. * indicates significance at 0.05 significance level.

Ng-Perron test aims to eliminate the distortion in the volume of the error term occurring in the PP unit root test. The Ng-Perron test has modified the PP unit root test and information criteria. While the basic hypothesis

(H_0) in the MZ_a and MZ_t tests in the Ng-Perron unit root test expresses the existence of a unit root, the basic hypothesis in the *MSB* and *MPT* tests shows that the series is stationary (Ng and Perron, 2001; Çetin and Saygın, 2019). According to the Ng-Perron test results applied to the level values and logarithmic values of the industrial production series, it was determined that the MIN_t series was stationary I(0) at the level in terms of both the level values and logarithmic values. The level values of the $TIPI_t$ and MI_t series were not found to be stationary at the level, but it is noticeable that the logarithmic transformations of the series ($LnTIPI_t$, $LnMI_t$) were stationary at the level. It has been estimated that the EGS_t and $LnEGS_t$ series are not stationary at the level and the series can become stationary at least at the first difference.

LS unit root test is calculated using two models, Model A and Model C. While Model A allows two breaks at the level, Model C allows two breaks at both the level and the trend. Based on previous studies, it has been observed that Model C offers superior results compared to Model A (Lean and Smyth, 2015; Berke et al., 2014; Karademir and Evci, 2020). In the LS unit root test, the basic hypothesis states the existence of a unit root under structural breaks, while the alternative hypothesis shows trend stationarity under structural breaks (Ertuğrul and Soytaş, 2013). According to the LS unit root test results presented in Table 2, it is seen that both the level values and the logarithmic values of the industrial production index are stationary I(0) at the level. According to the LS unit root test results applied to logarithmic values, findings were obtained that the industrial production index series were trend stationary under the structural breaks in the periods of $LnTIPI_t$ 2000:12-2005:2, $LnMIN_t$ 1997:12-2004:5, $LnMI_t$ 2001:2-2008:9 and $LnEGS_t$ 2000:10-2012:2.

CONCLUSION AND EVALUATION

In the study where the stationarity analysis of Türkiye's total industrial production and the indices of industrial sub-branches was performed using monthly time series covering the period 1986-2025, the Ng-Perron test, which is one of the traditional unit root tests, and the LS test, which is one of the unit root tests that take structural breaks into account, were applied. Before proceeding to the unit root analysis of the series, it was examined whether they provided the normal distribution assumption. According to the diagnostic statistical tests performed on the level values of the series, it was determined that they were not normally distributed and their natural logarithms (*ln*) were taken to make the distribution of the series closer to normal. As a result of the logarithmic transformation, it was observed that the skewness of the series decreased, therefore it was observed that the series had a distribution close to normal. After the series were made more ready for analysis, the unit root process was started. According to the Ng-Perron test results, the industrial production series were generally not found to be stationary at the level, but it was determined that the logarithmic values of the series were stationary at the level. According to the LS unit root test results, it was concluded that both level values and logarithmic values were trend stationary under structural breaks. In particular, it was determined that logarithmic values exhibited a more stationary structure than level values in terms of Ng-Perron unit root test results. The importance of logarithmic transformation in order for the series to have a normal distribution feature and to be made more stationary is understood from the analyses performed. The determination that the industrial production series adjusted for calendar and seasonal effects have a stationary structure means that the effects of shocks on the series are eliminated.

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Chapter 14

Homogeneous and Hybrid q-Mixture Forms of the Uma Distribution

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Abstract

Probability q-distributions provide a flexible and dynamic framework that generalizes classical probability distributions by introducing the q-parameter. This parameter allows for a broader class of probabilistic models, enriching both theoretical understanding and practical applications.

This study presents a comprehensive framework for the q-analogues of the Uma distribution and provides an in-depth analysis of the specific cases corresponding to the homogeneous and hybrid types. Although these distributions exhibit similar probability density functions, they exhibit significant differences in terms of parameter constraints, generative mechanisms, and underlying statistical properties.

The study encompasses the modeling of the q-analogues of both the probability density function and the cumulative distribution function, along with a detailed exploration of their shapes through rigorous mathematical analysis. Furthermore, the fundamental statistical and distributional characteristics of these distributions are thoroughly examined. Ultimately, these findings offer valuable insights for the continued exploration and application of q-Uma distributions across various fields.

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INTRODUCTION

Over the past few years, there has been a significant rise in the proposal of lifetime distributions for modeling lifetime data by various statisticians. In this context, the Uma distribution was introduced by Shanker in 2017, along with its probability density function (PDF) and cumulative distribution function (CDF), for the parameters $x \geq 0$ and $\lambda > 0$:

$$f(x;\lambda) = a\left(1 + x + x^3\right)e^{-\lambda x},\ a = \frac{\lambda^4}{\lambda^3 + \lambda^2 + 3!},$$

$$F(x;\lambda) = 1 - \left\{1 + \frac{a}{\lambda^4}\left[(\lambda x)^3 + 3(\lambda x)^2 + \left(\lambda^2 + 3!\right)\lambda x\right]\right\}e^{-\lambda x}.$$

In probability theory, the concept of *q*-distributions expands upon classical distributions, offering a wider array of possibilities. Key contributions to the theory of basic discrete *q*-distributions include the works of Dunkl, 1981, Crippa et al., 1997, Kupershmidt, 2000, Kemp, 2002, Charalambides, 2016, Kyriakoussis and Vamvakari, 2017, among others. Significant research on q-continuous distributions, including the Gaussian and generalized gamma *q*-distributions by Diaz et al., 2009, 2010, the Erlang *q*-distributions by Charalambides, 2016, the gamma and beta *q*-distributions by Boutouria et al., 2018, the Lindley *q*-distribution in two forms was introduced by Bouzida, 2023.

This research develops a comprehensive framework for the *q*-analogues of the Uma distribution, along with an extensive analysis of the particular cases that correspond to the homogeneous and hybrid types.

MATERIAL AND METHODS

This section outlines the principles of *q*-calculus, and *q*-probability theory. In this entire study, unless otherwise stated, it is assumed that $0 < q < 1$. Readers are referred to the relevant literature.

Definition 1 (Kac and Cheung, 2002). Let x, q be real numbers. The *q*-number $[x]_q$ is defined as

$$[x]_q = \frac{1 - q^x}{1 - q}.$$

For $n \in \mathbb{N}$, a natural *q*-number, it reduces to $[n]_q = \sum_{k=0}^{n-1} q^k$.

Definition 2 (Kac and Cheung, 2002). The *q*-Gauss binomial formula is given by

$$\left(x+y\right)_q^n=\sum_{k=0}^{n}\begin{bmatrix} n \\ k \end{bmatrix}_q q^{\binom{k}{2}}y^k x^{n-k},\ -\infty<x,y<\infty.$$

The q-binomial coefficients are provided for $k=0,1,\ldots,n$ by

$$\begin{bmatrix} n \\ k \end{bmatrix}_q=\frac{\left[n\right]_q!}{\left[n-k\right]_q!\left[k\right]_q!},\ \left[n\right]_{k,q}=\frac{\left[n\right]_q!}{\left[n-k\right]_q!},\ \left[n\right]_q! = \left[n\right]_q\left[n-1\right]_q\ldots\left[2\right]_q\left[1\right]_q.$$

Definition 3 (Kac and Cheung, 2002). The q-analogues of the exponential function are presented

$$E_q^{\tau}=\sum_{k=0}^{\infty}q^{\binom{k}{2}}\frac{\tau^k}{[k]_q!}=\prod_{k=0}^{\infty}\left(1+\left(1-q\right)q^k\tau\right),\quad \tau\in\mathbb{R},$$

$$e_q^{\tau}=\sum_{k=0}^{\infty}\frac{\tau^k}{[k]_q!}=\prod_{k=0}^{\infty}\frac{1}{\left(1-\left(1-q\right)q^k\tau\right)},\ \left|\tau\right|<\frac{1}{1-q}.$$

Definition 4 (Kac and Cheung, 2002). The q-derivative of f is defined as

$$D_q f\left(\tau\right)=\frac{f\left(q\tau\right)-f\left(\tau\right)}{q\tau-\tau}.$$

Definition 5 (Kac and Cheung, 2002). The well-known Jackson q-integral of f is given by

$$\int_0^b f\left(\tau\right)d_q\tau=\left(1-q\right)\sum_{n=0}^{\infty}q^n bf\left(q^n b\right),\ b>0,$$

Definition 6 (Kac and Cheung, 2002). The generalized q-integral is given by

$$\int_{-\infty}^{\infty} f\left(\tau\right)d_q\tau=\left(1-q\right)\sum_{n\in\mathbb{Z}}q^n f\left(q^n\right).$$

Definition 7 (Kac and Cheung, 2002, De Sole and Kac, 2003). The q-analogues of the gamma functions are given for $\alpha>0$, $n\in\mathbb{N}$

$$\Gamma_q\left(\alpha\right)=\int_0^{[\infty]_q}x^{\alpha-1}E_q^{-qx}d_qx,\ \gamma_q\left(\alpha\right)=\int_0^{\infty}x^{\alpha-1}e_q^{-x}d_qx.$$

Identities derived from the q-gamma functions can be obtained

$$\Gamma_q(\alpha+n)=[\alpha]_{n,q}\Gamma_q(\alpha) \qquad \gamma_q(\alpha+n)=q^{-\binom{\alpha+n}{2}}[\alpha]_{n,q}\gamma_q(\alpha)$$

$$\Gamma_q(n+1)=[n]_q! \qquad \gamma_q(n+1)=q^{-\binom{n+1}{2}}[n]_q!$$

Definition 8 (Vamvakari, 2023). A random variable X is considered q-continuous if there exists a non-negative function $f_q^X(x)$ for $x \geq 0$ such that

$$P\{a < X \leq b\} = \int_a^b f_q^X(x)\, d_q x.$$

The q-cumulative distribution function (q-CDF) of X. is defined for $x > 0$

$$F_q^X(x) = P(X \leq x) = \int_0^x f_q^X(u)\, d_q u,$$

satisfying the relation $P(\alpha < X \leq \beta) = F_q^X(\beta) - F_q^X(\alpha)$. Then,

$$f_q(x) = D_q F_q(x) = \frac{F_q^X(x) - F_q^X(qx)}{(1-q)x} = \frac{P(qx \leq X \leq x)}{(1-q)x}, \quad (q \neq 1 \;).$$

Definition 9 (Vamvakari, 2023). Let $\mathbb{E}_q\left|X^r\right| < \infty$ for all positive integers r. Then,

$$\mu_q^{(r)} = \mathbb{E}_q(X^r) = \int_0^\infty x^r \; f_q^X(x)\, d_q x, \;\; \mathbb{E}_q(X) = \mu_q, \;\; \mathbb{V}_q(X) = \mu_q^{(2)} - \mu_q^2.$$

RESULTS

This section presents the Uma q-distribution, including its homogeneous and hybird forms.

Modeling of the Uma q-distribution

A q-mixture model of the Uma distribution is a q-probabilistic parametric model composed of multiple component Uma$q$-distributions combined with certain parametric weights. The general form of the q-parametric mixture model of the Uma distribution is for $x \geq 0$. characterized by

$$f_q^{i_n}(x;\lambda) = \sum_{\alpha \in D} p_q^i(\alpha)\, g_q^{C_n}(x;\alpha,\lambda), \;\; (i = I, II;\; n = 1,2,\ldots,8)$$

where

$\alpha \in D$: Parameter value in the set of components $D=\{1,2,4\}$

$p_q^i(\alpha)$: Mixing parametric proportion of the α-th component, $p_q^i(\alpha)\geq 0$, $\sum_{\alpha\in D} p_q^i(\alpha)=1$ and

$$p_q^I(\alpha)=\frac{\Gamma_q(\alpha)\lambda^{1-\alpha}}{\sum_{k\in D}\Gamma_q(\alpha)\lambda^{1-\alpha}},\ p_q^{II}(\alpha)=\frac{\gamma_q(\alpha)\lambda^{1-\alpha}}{\sum_{k\in D}\gamma_q(\alpha)\lambda^{1-\alpha}}$$

$g_q^{C_n}(x;\alpha,\lambda)$: Gamma *q*-PDF of the α-th component and

$$g_q^I(x;\alpha,\lambda)=\frac{\lambda^\alpha}{\Gamma_q(\alpha)}x^{\alpha-1}E_q^{-q\lambda x}1_{\left[0,[\infty]_q\right]}(x),\ g_q^{II}(x;\alpha,\lambda)=\frac{\lambda^\alpha}{\gamma_q(\alpha)}x^{\alpha-1}e_q^{-\lambda x}1_{[0,\infty)}(x)$$

C_n : Selecting function the appropriate *q* or $1/q$ -PDF.

As $q\to 1$, the *q*-PDF $f_q^{i_n}$ converges to the ordinary Uma PDF. The function $f_q^{i_n}$ satisfies the necessary conditions to qualify as a *q*-PDF. Furthermore, the corresponding *q*-CDF is:

$$F_q^{i_n}(x;\lambda)=\sum_{\alpha\in D} p_q^i(\alpha)G_q^{C_n}(x;\alpha,\lambda),\ (i=I,II;\ n=1,2,...,8)$$

where $G_q^i(x,\alpha,\lambda)$ is the gamma *q*-CDF of the α-th component, and

$$G_q^I(x,\alpha,\lambda)=\frac{\Gamma_q(\alpha,\lambda x)}{\Gamma_q(\alpha)},\ G_q^{II}(x,\alpha,\lambda)=\frac{\gamma_q(\alpha,\lambda x)}{\gamma_q(\alpha)}$$

The *q*-parametric mixture model of the Uma distribution is a broad family of distributions that includes the original *q*-formulation, the reciprocal formulation based on $1/q$, as well as their hybrid combinations. Let $p_q^i(\alpha)$ and $g_q^i(x;\alpha,\lambda)$ be $p_q^{i(\alpha)}$ and $g_q^{i(\alpha)}$, and let us define the notations:

$$a_q^I=\frac{\lambda^4}{\lambda^3+\lambda^2+[3]_q!},\ a_q^{II}=\frac{\lambda^4}{\lambda^3+q^{-1}\lambda^2+q^{-6}[3]_q!}$$

Homogeneous Type of the Uma *q*-distribution

This section introduces the homogeneous Uma *q*-distribution and presents its distributional charachteristics.

The homogeneous Uma q-distribution can be constructed via the convex combination of $p_q^i(\alpha)$ and $g_q^i(x;\alpha,\lambda)$ for $i=I,II$ as summarized in Table 1.

Table 1. Convex combinations for the homogeneous Uma q-PDF

n	The first type of HoUma q-PDF $f_q^{I_n}(x;\lambda)$	The second type of HoUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$p_q^{I(1)}g_q^{I(1)}+p_q^{I(2)}g_q^{I(2)}+p_q^{I(4)}g_q^{I(4)}$	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{I(4)}$
2	$p_q^{I(1)}g_q^{II(1)}+p_q^{I(2)}g_q^{II(2)}+p_q^{I(4)}g_q^{II(4)}$	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{II(4)}$

All variants of the homogeneous Uma q-distribution, $f_q^{i_n}(x;\lambda)(i=,II,n=1,2)$, are presented in Table 2, with normalizing constants a_q^I and a_q^{II}.

Table 2. The two types of homogeneous Uma q-PDF

n	The first type of HoUma q-PDF $f_q^{I_n}(x;\lambda)$	The second type of HoUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$a_q^I\left(1+x+x^3\right)E_q^{-q\lambda x}$	$a_q^{II}\left(1+q^{-1}x+q^{-6}x^3\right)E_q^{-q\lambda x}$
2	$a_q^I\left(1+qx+q^6x^3\right)e_q^{-\lambda x}$	$a_q^{II}\left(1+x+x^3\right)e_q^{-\lambda x}$

The graphs below illustrate the homogeneous Uma q-PDF for $\lambda=0.5$ and $\lambda=2$ with varying values of the q-parameter as outlined in Table 2.

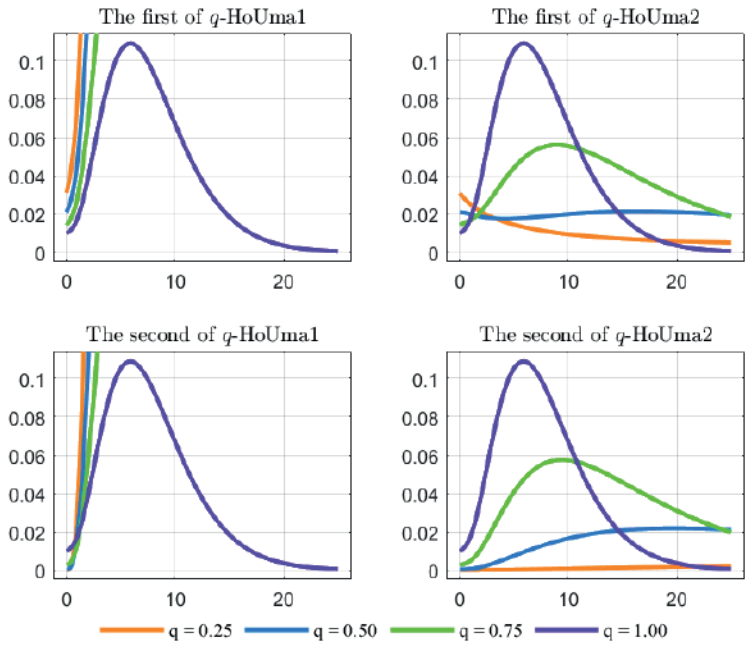

Figure 1. Graphs of the homogeneous Uma q-PDF for $\lambda = 0.5$ and varying values of the q-parameter

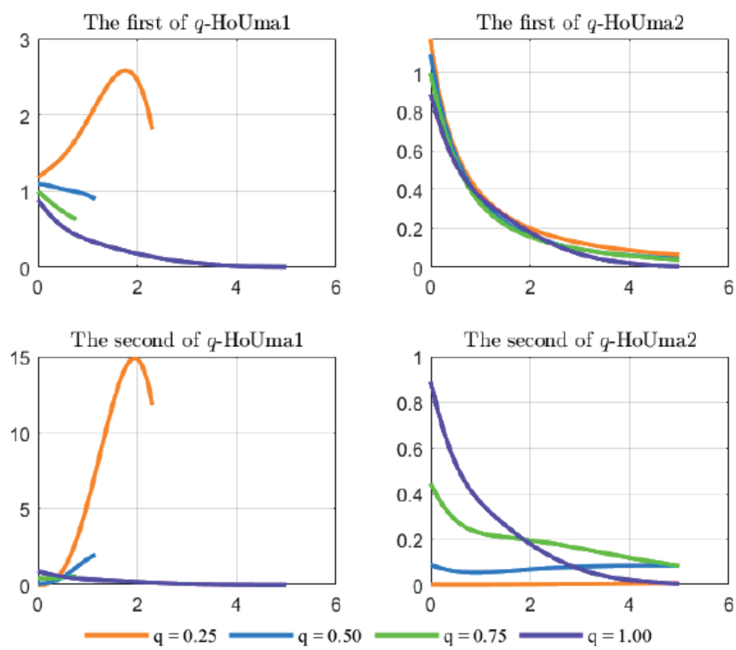

Figure 2. Graphs of the homogeneous Uma q-PDF for $\lambda = 2$ and varying values of the q-parameter

The plots above illustrate two types of q-HoUma distributions (for $m=1,2$), representing the q-PDFs of the homogeneous q-Uma distribution. It is observed that increasing the parameter q enhances the convexity of the model in the case of q-HoUma1, while it increases the concavity in the case of q-HoUma2. Moreover, all q-Uma distributions converge to their standard form as $q \to 1$.

The first type of homogeneous Uma q-CDF, q-moment, and q-related measures are presented as outlined in Table 3.

Table 3. The first type of homogeneous Uma q-CDF, q-Moment and related measures

n	The first type of HoUma q-CDF $F_q^{I_n}(x;\lambda)$
1	$1-\left\{1+\frac{a_q^I}{\lambda^4}\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+\lambda^3\tau\right]\right\}E_q^{-\lambda\tau}$
2	$1-\left\{1+\frac{a_q^I}{\lambda^4}\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+\lambda^3\tau\right]\right\}e_q^{-\lambda x}$
n	**The first type of HoUma q-Moment $\mu_q^{(r)(I_n)}$**
1	$a_q^I\left([r]_q!\lambda^{-(r+1)}+[r+1]_q!\lambda^{-(r+2)}+[r+4]_q!\lambda^{-(r+5)}\right)$
2	$a_q^I\left(q^{-\binom{r+1}{2}}[r]_q!\lambda^{-(r+1)}+q^{1-\binom{r+2}{2}}[r+1]_q!\lambda^{-(r+2)}+q^{6-\binom{r+5}{2}}[r+4]_q!\lambda^{-(r+5)}\right)$
n	**The first type of HoUma q-expected value**
1	$a_q^I\left(\lambda^{-2}+[2]_q!\lambda^{-3}+[5]_q!\lambda^{-6}\right)$
2	$a_q^I\left(q^{-1}\lambda^{-2}+q^{-2}[2]_q!\lambda^{-3}+q^{-9}[5]_q!\lambda^{-6}\right)$
n	**The first type of HoUma q-variance**
1	$a_q^I\left([2]_q!\lambda^{-3}+[3]_q!\lambda^{-4}+[6]_q!\lambda^{-7}\right)-\left(a_q^I\right)^2\left(\lambda^{-2}+[2]_q!\lambda^{-3}+[5]_q!\lambda^{-6}\right)^2$
2	$a_q^I\left(q^{-1}[2]_q!\lambda^{-3}+q^{-2}[3]_q!\lambda^{-4}+q^{-9}[6]_q!\lambda^{-7}\right)-\left(a_q^I\right)^2\left(q^{-1}\lambda^{-2}+q^{-2}[2]_q!\lambda^{-3}+q^{-9}[5]_q!\lambda^{-6}\right)^2$

As shown in Table 4, the second type of homogeneous Uma q-CDF, q-moment, and q-related measures are given by:

Table 4. The second type of homogeneous Uma q-CDF, q-Moment and related measures

n	The second type of HoUma q-CDF $F_q^{II_n}(x;\lambda)$
1	$1-\left\{1+\frac{a_q^{II}}{\lambda^4}\left[q^{-6}\left((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau\right)+q^{-1}\lambda^3\tau\right]\right\}E_q^{-\lambda\tau}$
2	$1-\left\{1+\frac{a_q^{II}}{\lambda^4}\left[q^{-6}\left(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau\right)+q^{-1}\lambda^3\tau\right]\right\}e_q^{-\lambda\tau}$
n	The second type of HoUma q-Moment $\mu_q^{(r)(II_n)}$
1	$a_q^{II}\left([r]_q!\lambda^{-(r+1)}+q^{-1}[r+1]_q!\lambda^{-(r+2)}+q^{-6}[r+4]_q!\lambda^{-(r+5)}\right)$
2	$a_q^{II}\left(q^{-\binom{r+1}{2}}[r]_q!\lambda^{-(r+1)}+q^{-\binom{r+2}{2}}[r+1]_q!\lambda^{-(r+2)}+q^{-\binom{r+5}{2}}[r+4]_q!\lambda^{-(r+5)}\right)$
n	The second type of HoUma q-expected value
1	$a_q^{II}\left(\lambda^{-2}+q^{-1}[2]_q!\lambda^{-3}+q^{-6}[5]_q!\lambda^{-6}\right)$
2	$a_q^{II}\left(q^{-1}\lambda^{-2}++q^{-3}[2]_q!\lambda^{-3}+q^{-15}[5]_q!\lambda^{-6}\right)$
n	The second type of q-variance
1	$a_q^{II}\left([2]_q!\lambda^{-3}+q^{-1}[3]_q!\lambda^{-4}+q^{-6}[6]_q!\lambda^{-7}\right)-\left(a_q^{II}\right)^2\left(\lambda^{-2}+q^{-1}[2]_q!\lambda^{-3}+q^{-6}[5]_q!\lambda^{-6}\right)^2$
2	$a_q^{II}\left(q^{-1}[2]_q!\lambda^{-3}++q^{-3}[3]_q!\lambda^{-4}+q^{-15}[6]_q!\lambda^{-7}\right)-\left(a_q^{II}\right)^2\left(q^{-1}\lambda^{-2}++q^{-3}[2]_q!\lambda^{-3}+q^{-15}[5]_q!\lambda^{-6}\right)^2$

Hybrid Type of the Uma *q*-distribution

This section introduces the hybrid Uma q-distribution and presents its distributional charachteristics.

The hybrid Uma q-distribution can be constructed via the convex combination of $p_q^i(\alpha)$ and $g_q^i(x;\alpha,\lambda)$ for $i=I,II$ as summarized in Table 5.

Table 5. Convex combinations for the hybrid Uma q-PDFs

n	The first type of HyUma q-PDF $f_q^{I_n}(x;\lambda)$	The second type of HyUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$p_q^{I(1)}g_q^{I(1)}+p_q^{I(2)}g_q^{I(2)}+p_q^{I(4)}g_q^{II(4)}$	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{II(4)}$
2	$p_q^{I(1)}g_q^{I(1)}+p_q^{I(2)}g_q^{II(2)}+p_q^{I(4)}g_q^{I(4)}$	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{I(4)}$
3	$p_q^{I(1)}g_q^{I(1)}+p_q^{I(2)}g_q^{II(2)}+p_q^{I(4)}g_q^{II(4)}$	$p_q^{II(1)}g_q^{I(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{II(4)}$
4	$p_q^{I(1)}g_q^{II(1)}+p_q^{I(2)}g_q^{I(2)}+p_q^{I(4)}g_q^{II(4)}$	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{II(4)}$
5	$p_q^{I(1)}g_q^{II(1)}+p_q^{I(2)}g_q^{II(2)}+p_q^{I(4)}g_q^{I(4)}$	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{II(2)}+p_q^{II(4)}g_q^{I(4)}$
6	$p_q^{I(1)}g_q^{II(1)}+p_q^{I(2)}g_q^{I(2)}+p_q^{I(4)}g_q^{I(4)}$	$p_q^{II(1)}g_q^{II(1)}+p_q^{II(2)}g_q^{I(2)}+p_q^{II(4)}g_q^{I(4)}$

All variants of the hybrid Uma q-PDF, $f_q^{i_n}(x;\lambda)\left(i=,II,n=1,2,\ldots,6\right)$, are presented in Table 2, with normalizing constants a_q^I and a_q^{II}.

Table 6. The hybrid Uma q-PDFs

n	The first type of HyUma q-PDF $f_q^{I_n}(x;\lambda)$	The second type of HyUma q-PDF $f_q^{II_n}(x;\lambda)$
1	$a_q^I\left\{(1+x)E_q^{-q\lambda x}+q^6x^3e_q^{-\lambda x}\right\}$	$a_q^{II}\left\{(1+q^{-1}x)E_q^{-q\lambda x}+x^3e_q^{-\lambda x}\right\}$
2	$a_q^I\left\{(1+x^3)E_q^{-q\lambda x}+qxe_q^{-\lambda x}\right\}$	$a_q^{II}\left\{(1+q^{-6}x^3)E_q^{-q\lambda x}+xe_q^{-\lambda x}\right\}$
3	$a_q^I\left\{E_q^{-q\lambda x}+(qx+q^6x^3)e_q^{-\lambda x}\right\}$	$a_q^{II}\left\{E_q^{-q\lambda x}+(x+x^3)e_q^{-\lambda x}\right\}$
4	$a_q^I\left\{xE_q^{-q\lambda x}+(1+q^6x^3)e_q^{-\lambda x}\right\}$	$a_q^{II}\left\{q^{-1}xE_q^{-q\lambda x}+(1+x^3)e_q^{-\lambda x}\right\}$
5	$a_q^I\left\{x^3E_q^{-q\lambda x}+(1+x)e_q^{-\lambda x}\right\}$	$a_q^{II}\left\{q^{-6}x^3E_q^{-q\lambda x}+(1+x)e_q^{-\lambda x}\right\}$
6	$a_q^I\left\{(x+x^3)E_q^{-q\lambda x}+e_q^{-\lambda x}\right\}$	$a_q^{II}\left\{(q^{-1}x+q^{-6}x^3)E_q^{-q\lambda x}+e_q^{-\lambda x}\right\}$

The hybrid configurations of the Uma q-PDF for $\lambda = 0.5$ and $\lambda = 2$ with varying values of the q-parameter as outlined in Table 3 and 5. are depicted in Figures 2 and 3.

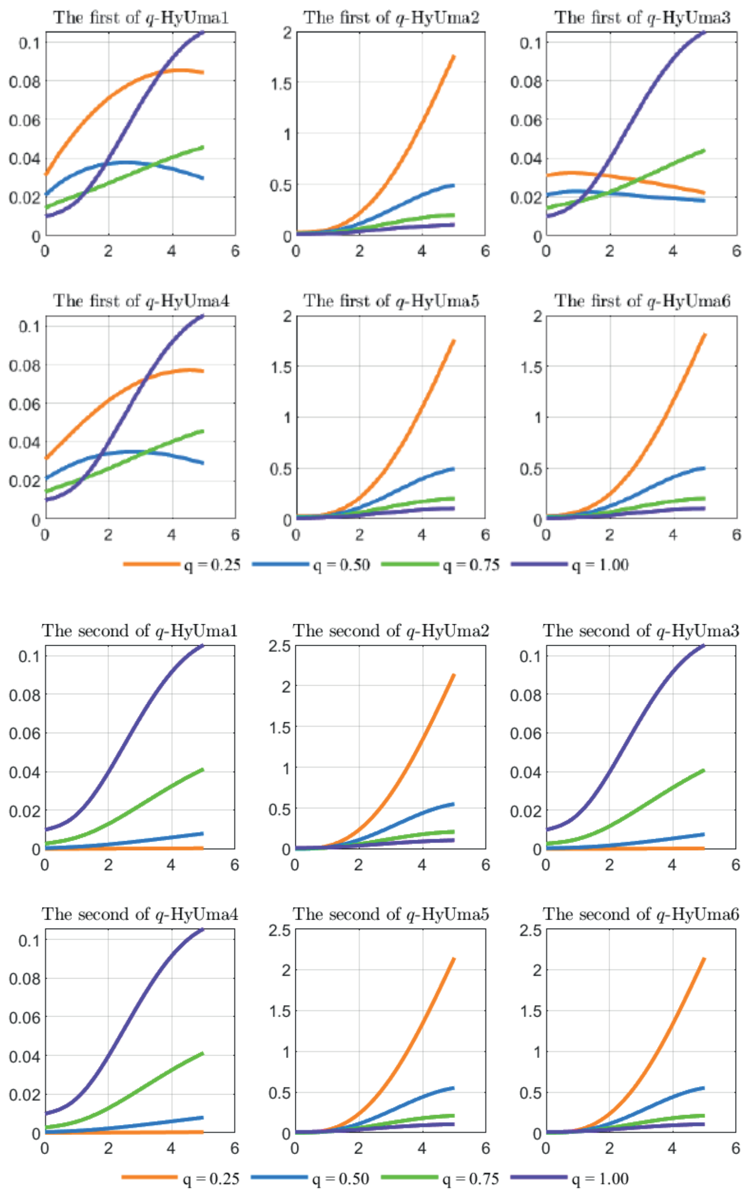

Figure 2. Graphs of the hybrid Uma q-PDF for $\lambda = 0.5$ ***and varying values of the q-parameter***

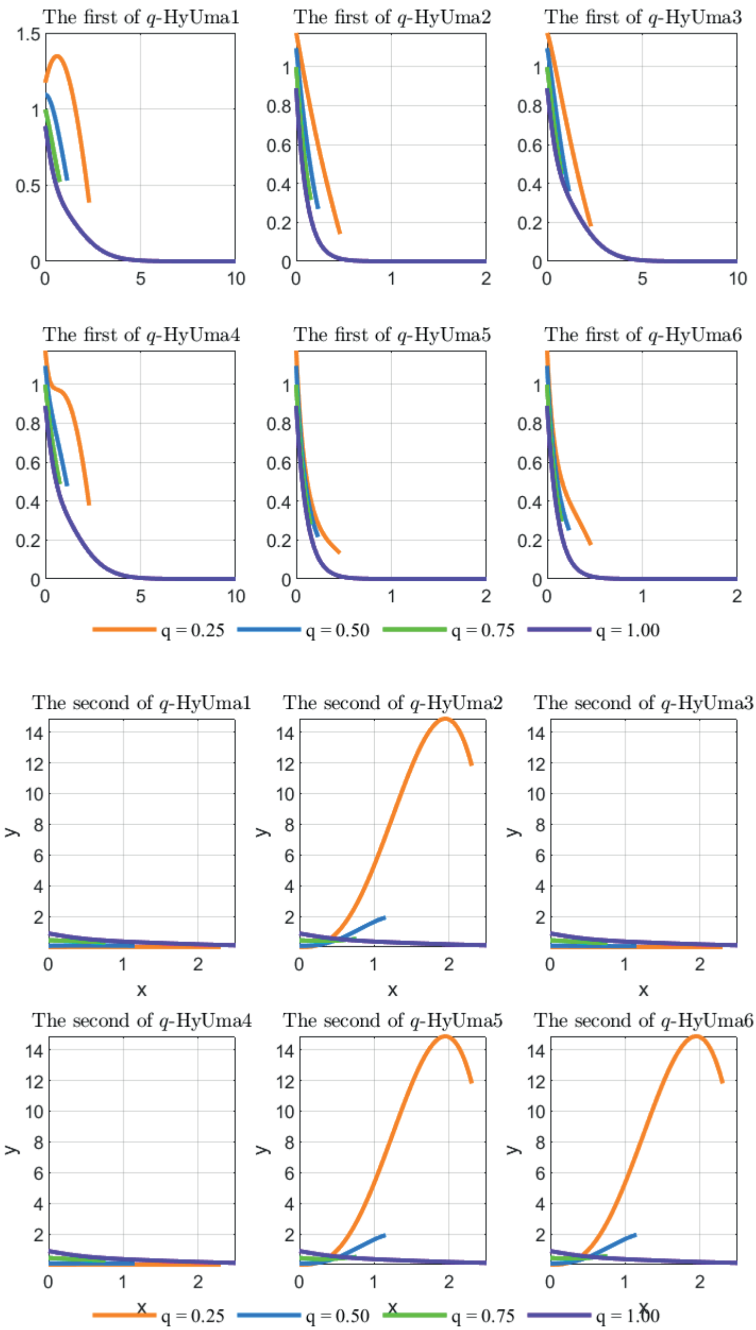

Figure 3. Graphs of the hybrid Uma q-PDF for $\lambda = 2$ and varying values of the q-parameter

Two hybrid q-Uma models, each consisting of six q-PDFs, are introduced. The 2nd, 5th, and 6th q-PDFs exhibit similar patterns, while the 1st, 3rd, and 4th form another group. This similarity arises from the multiplication of the q-exponential function $E_q^{-q\lambda x}$ by x^3.

The two types of homogeneous Uma q-CDF are presented as outlined in Table 4.

Table 4. The hybrid Uma q-CDFs

n	The first type of HoUma q-CDF $F_q^{I_n}(x;\lambda)$
1	$1-\frac{a_q^I}{\lambda^4}\left\{\left[\lambda^3+\lambda^2(1+\lambda\tau)\right]E_q^{-\lambda\tau}+\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right]e_q^{-\lambda\tau}\right\}$
2	$1-\frac{a_q^I}{\lambda^4}\left\{\lambda^2(1+q\lambda\tau)e_q^{-\lambda\tau}+\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!+\lambda^3\right]E_q^{-\lambda\tau}\right\}$
3	$1-\frac{a_q^I}{\lambda^4}\left\{\lambda^3E_q^{-\lambda\tau}+\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!+\lambda^2(1+\lambda\tau)\right]e_q^{-\lambda\tau}\right\}$
4	$1-\frac{a_q^I}{\lambda^4}\left\{\lambda^2(1+\lambda\tau)E_q^{-\lambda\tau}+\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!+\lambda^3\right]e_q^{-\lambda\tau}\right\}$
5	$1-\frac{a_q^I}{\lambda^4}\left\{\left[\lambda^3+\lambda^2(1+q\lambda\tau)\right]e_q^{-\lambda\tau}+\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right]E_q^{-\lambda\tau}\right\}$
6	$1-\frac{a_q^I}{\lambda^4}\left\{\lambda^3e_q^{-\lambda\tau}+\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!+\lambda^2(1+\lambda\tau)\right]E_q^{-\lambda\tau}\right\}$
n	The second type of HoUma q-CDF $F_q^{II_n}(x;\lambda)$
1	$1-\frac{a_q^{II}}{\lambda^4}\left\{\left[\lambda^3+q^{-1}\lambda^2(1+\lambda\tau)\right]E_q^{-\lambda\tau}+q^{-6}\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right]e_q^{-\lambda\tau}\right\}$
2	$1-\frac{a_q^{II}}{\lambda^4}\left\{q^{-1}\lambda^2(1+q\lambda\tau)e_q^{-\lambda\tau}+\left[q^{-6}\left((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right)+\lambda^3\right]E_q^{-\lambda\tau}\right\}$
3	$1-\frac{a_q^{II}}{\lambda^4}\left\{\lambda^3E_q^{-\lambda\tau}+\left[q^{-6}\left(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right)+q^{-1}\lambda^2(1+\lambda\tau)\right]e_q^{-\lambda\tau}\right\}$
4	$1-\frac{a_q^{II}}{\lambda^4}\left\{q^{-1}\lambda^2(1+\lambda\tau)E_q^{-\lambda\tau}+\left[q^{-6}\left(q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right)+\lambda^3\right]e_q^{-\lambda\tau}\right\}$
5	$1-\frac{a_q^{II}}{\lambda^4}\left\{\left[\lambda^3+q^{-1}\lambda^2(1+q\lambda\tau)\right]e_q^{-\lambda\tau}+q^{-6}\left[(\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right]E_q^{-\lambda\tau}\right\}$

n	The first type of HoUma q-CDF $F_q^{I_n}(x;\lambda)$
1	$1-\frac{a_q^I}{\lambda^4}\left\{\left[\lambda^3+\lambda^2(1+\lambda\tau)\right]E_q^{-\lambda\tau}+\left[q^3(\lambda\tau)^3+q[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right]e_q^{-\lambda\tau}\right\}$
6	$1-\frac{a_q^{II}}{\lambda^4}\left\{\lambda^3 e_q^{-\lambda\tau}+\left[q^{-6}\left((\lambda\tau)^3+[3]_q(\lambda\tau)^2+[3]_q!\lambda\tau+[3]_q!\right)+q^{-1}\lambda^2(1+\lambda\tau)\right]E_q^{-\lambda\tau}\right\}$

Furthermore, the derivation of the hybrid q-moment and the corresponding statistical measures can be carried out in an analogous manner for the homogeneous type.

DISCUSSION AND CONCLUSION

The evolution of q-distributions represents a natural progression in the development of q-calculus. q-calculus serves as a parametric generalization of classical calculus, with the classical framework being recovered in the limit as $q \to 1$.

In this paper, we introduce the Uma q-distribution, and and provide a detailed analysis of the structural and statistical properties, including its modeling, shape, moment of the specific cases corresponding to the homogeneous and hybrid types.

Our findings suggest that the proposed q-distribution holds significant promise and may have widespread applications across various fields. In future research, we aim to explore the finite mixture and compound q-distribution.

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Conflict of Interest

The authors have declared that there is no conflict of interest.

Author Contributions

Both authors contributed equally to the finalization of this paper.

Chapter 15

Evaluating the Economic Landscape of Turkey's Cosmetics Industry (2020–2022)

Tuğçe Yıldırım Dal[1]

Dursun Yılmaz[2]

Abstract

This study aims to analyze the economic performance of the Turkish cosmetics industry during the 2020–2022 period in terms of exports, imports, and market dynamics. Based on quantitative data analysis, the research utilizes updated datasets from the Ministry of Trade, the Turkish Statistical Institute (TURKSTAT), and international trade databases. The findings reveal that the sector recorded an average annual export growth rate of 18.7%. Key determinants of the industry's competitive strength include the asymmetric distribution across product categories (e.g., shaving products accounting for 24.7%), geographic market dependency (notably Iraq and the EU with a combined share of 39.2%), and low R&D intensity (1.8% of total revenue). The study recommends policy strategies focused on branding, green transformation, and market diversification to enable Turkey to ascend within the global cosmetics value chain.

INTRODUCTION

The cosmetics industry stands as one of the most dynamic and steadily growing sectors in global trade in terms of market volume. According to data from Grand View Research (2023), the global cosmetics market reached a value of USD 571.1 billion in 2022 and is projected to continue expanding with a compound annual growth rate (CAGR) of 4.8% between 2023 and 2030. This growth trajectory is shaped by a range of multidimensional

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factors, including changes in consumer behavior, technological innovations, and the accelerating effects of globalization.

Turkey has been gaining increasing significance in this global landscape due to its strategic geographic location and young, dynamic population structure. Particularly in recent years, the striking increase in the export performance of the Turkish cosmetics sector has highlighted the country's potential to become a regional production hub. According to the Ministry of Trade (2023), Turkey's cosmetics exports grew by 40.6% between 2020 and 2022, rising from USD 953 million to USD 1.34 billion.

The primary objective of this study is to systematically analyze the economic performance of the Turkish cosmetics industry over the 2020–2022 period and to identify the main factors influencing the sector's global competitiveness. Specifically, the study seeks to address the following key research questions: (1) What have been the main drivers of growth in the Turkish cosmetics industry over the past three years? (2) Which product categories have stood out in exports, and how do they impact the sector's competitive strength? (3) What strategic steps should be taken to enhance Turkey's current position in the global cosmetics market?

The significance of this study lies in its comprehensive evaluation of Turkey's performance in the cosmetics industry and its ability to offer strategic policy recommendations for both policymakers and industry stakeholders. Furthermore, this research aims to make a meaningful contribution to the literature by providing insights into the positioning of emerging economies within the global cosmetics market.

LITERATURE REVIEW

The literature on the economic analysis of the cosmetics industry is generally shaped around three main axes: global market dynamics, the role of emerging economies in the sector, and sector-specific developments in the context of Turkey.

Global Dynamics of the Cosmetics Industry

Over the past two decades, the cosmetics industry has emerged as one of the most consistently growing sectors in global trade. According to analyses by McKinsey & Company (2022), the global cosmetics market recorded an average annual growth rate of 5.2% between 2000 and 2020. This growth has been largely driven by the expansion of the middle class in the Asia-Pacific region and the widespread adoption of digital channels. Euromonitor (2023) reports that demand for premium cosmetic products increased by 7.4% in 2022.

Emerging Economies and the Cosmetics Sector

The share of emerging economies in the global cosmetics market has been steadily rising. World Bank (2021) data show that their share of global cosmetics consumption increased from 32% in 2010 to 41% in 2020. However, Erkan and Sarıoğlu (2022) highlight that many of these economies still lag behind advanced countries in terms of technological development and brand creation. In this regard, R&D investment and innovation capacity remain key determinants of competitive advantage.

Studies on the Turkish Cosmetics Industry

Academic interest in the performance of the Turkish cosmetics industry has been growing in recent years. According to data from the Ministry of Trade (2023), Turkish cosmetics exports reached USD 1.34 billion in 2022, with the Middle East and Europe being the primary export destinations. Nevertheless, Demirtaş (2021) points out that approximately 90% of cosmetics manufactured in Turkey are produced for international brands rather than local ones. This situation underlines the ongoing challenges in branding and value-added production.

Definition and Scope of the Cosmetics Industry

The cosmetics industry refers to a multidimensional economic field involving the production, distribution, and marketing of products designed to enhance appearance, maintain hygiene, alter body odor, and promote skin health. According to the European Union's Cosmetics Regulation (EC 1223/2009), cosmetic products are defined as "substances or mixtures intended to be placed in contact with the external parts of the human body (epidermis, hair, nails, lips, and external genital organs) or with the teeth and the mucous membranes of the oral cavity, with a view exclusively or mainly to cleaning them, perfuming them, changing their appearance, protecting them, keeping them in good condition, or correcting body odors." This definition is also adopted by the Turkish Medicines and Medical Devices Agency (TMMDA).

The sector encompasses not only personal care products but also dermocosmetics, natural and organic cosmetics, aesthetic-focused products, and items used in professional beauty salons. In this sense, the industry includes both business-to-consumer (B2C) and business-to-business (B2B) production and distribution networks.

In this study, the cosmetics sector is evaluated based on international trade data corresponding to product groups classified under HS Codes 3303 to 3307. These groups include perfumes and toilet waters (3303), makeup and skincare products (3304), hair care products (3305), oral and dental hygiene products (3306), and shaving products, deodorants, and other body care items (3307). The sectoral analysis considers both foreign trade data and domestic consumption trends related to these product categories.

The scope of the cosmetics industry is shaped not only by economic indicators but also by transformations in consumer preferences (e.g., sustainability, vegan ingredients, environmentally friendly packaging) and advancements in production technologies. Therefore, sectoral evaluations must incorporate not only financial indicators but also socio-cultural trends and the regulatory framework.

Research Gap and Contribution of the Study

A review of the existing literature reveals that there is a limited number of comprehensive studies that examine the economic performance of the Turkish cosmetics sector using post-2020 data. This study seeks to address this gap by providing a detailed analysis of sectoral developments in the post-pandemic period and evaluating Turkey's position within global value chains. Additionally, the study offers original contributions by proposing strategic policy recommendations aimed at enhancing the sector's international competitiveness.

MATERIAL AND METHODS

This research is a descriptive study that aims to analyze the economic performance of the Turkish cosmetics industry between 2020 and 2022 through quantitative data. A cross-sectional time series analysis design was adopted to reveal sectoral trends over time. This method enables a comparative evaluation of economic indicators within a specific time frame (Yin, 2018). Such an approach provides a solid foundation for understanding the dynamics of the cosmetics sector and making forward-looking inferences.

The data collection process was conducted in three main stages. In the first stage, raw data were systematically compiled from publicly available primary sources, including the 2023 Cosmetics Sector Report published by the Ministry of Trade of the Republic of Turkey, export data provided by the Turkish Exporters Assembly (TİM), and the industrial production index published by the Turkish Statistical Institute (TURKSTAT).

In the second stage, detailed import and export statistics were obtained from international trade databases such as ITC Trademap and UN Comtrade. These data were analyzed through country- and product-based comparisons. In addition, reliable secondary sources such as category-based market analyses provided by Euromonitor International were utilized to enrich the analysis.

In the third and final stage, content analysis of sector-related policy documents, strategic reports, and regulatory texts was conducted to derive qualitative insights that complement the quantitative findings. This multi-layered approach is consistent with document-based research methodologies widely employed in the social sciences (Bowen, 2009; Flick, 2014).

The analytical techniques used in this study comprise both quantitative and qualitative methods. The quantitative analysis includes calculations of compound annual growth rate (CAGR), market share analysis by product category, export concentration index, and correlation analysis to evaluate the relationship between Turkey's GDP growth and sectoral performance. These analyses are significant for assessing the export capacity and internal dynamics of the industry (Bryman, 2016). In terms of qualitative methods, content analysis was applied to sector-related policy documents and reports, and a SWOT analysis was conducted to examine the competitive capacity of the Turkish cosmetics sector.

The scope of the study is limited to cosmetic products classified under HS Codes 3303–3307 (Harmonized System Codes). The research covers only official data from the 2020–2022 period and focuses on Turkey's top ten export markets for cosmetic products. However, the study has certain limitations. Due to the unavailability of official data for 2023 and beyond, these years were excluded from the analysis. Additionally, access to data on informal trade flows and product-level profitability was restricted.

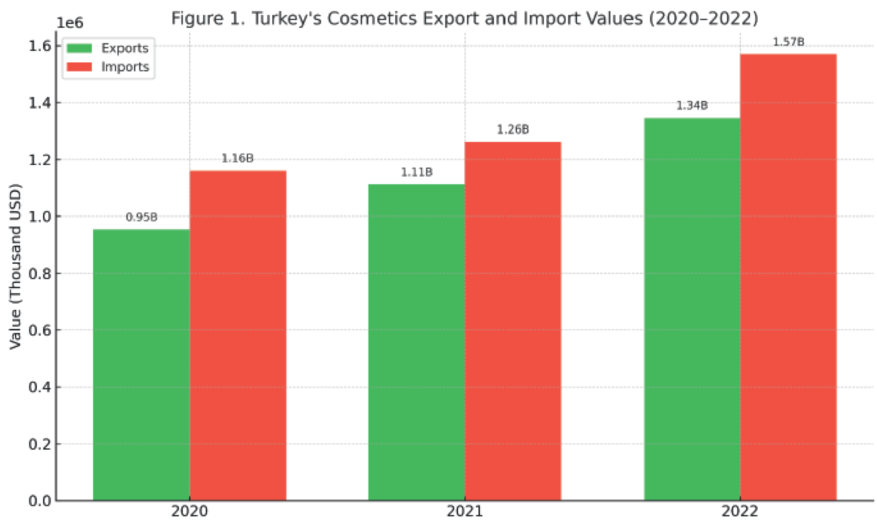
As part of the ethical considerations, all data used in this study were obtained from publicly accessible and credible sources. No data manipulation was performed. Throughout the research process, principles of academic integrity were strictly observed, and all sources were cited in accordance with APA 7 referencing guidelines.

RESULTS

1. Export Growth and Import Dependency in the Cosmetics Sector (2020–2022)

Between 2020 and 2022, Turkey's cosmetics sector recorded a significant upward trend in foreign trade indicators. According to official data from the Ministry of Trade (2023) and the Turkish Statistical Institute (TURKSTAT, 2023), the sector's export value stood at USD 953 million in 2020 and rose by 40.6% to reach USD 1.34 billion by 2022. This increase corresponds to a robust compound annual growth rate (CAGR) of 18.7%. During the same period, imports increased from USD 1.16 billion to USD 1.57 billion, marking a 24.5% rise. Consequently, as of 2022, the sector recorded a foreign trade deficit of approximately USD 225.6 million.

This situation indicates that despite the growth in export capacity, the sector continues to demonstrate structural dependency on imported inputs for production. Notably, the import composition is dominated by fragrance compounds used in manufacturing and high value-added skincare products, while exports are largely composed of shaving products, hair preparations, and perfumes.



Source: TURKSTAT, 2023; Ministry of Trade, 2023.

Figure 1. Turkey's Cosmetics Sector Export and Import Values (2020–2022)
Between 2020 and 2022, exports increased by 40.6% and imports by 24.5%. During this period, the net trade deficit reached –USD 225.6 million, reflecting the sector's ongoing dependency on imported inputs.

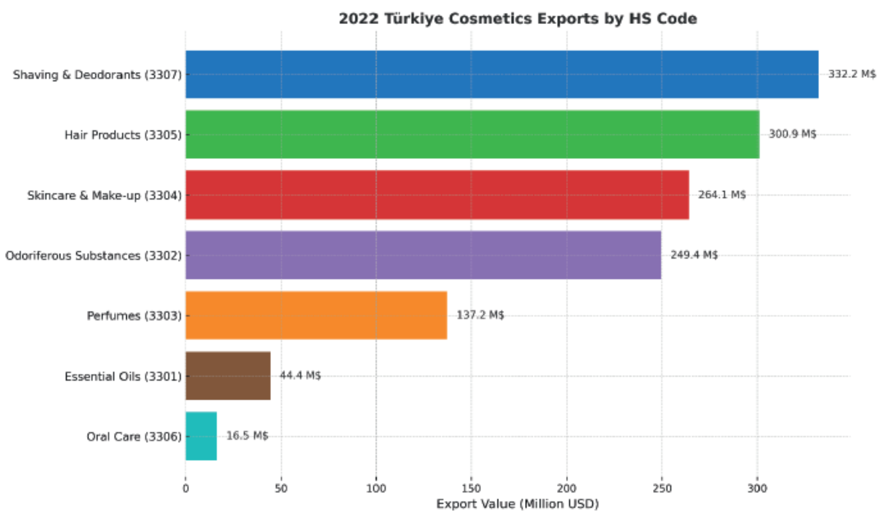
2. Asymmetric Distribution Across Product Categories

The export performance of Turkey's cosmetics industry exhibits a marked asymmetry across product categories. As illustrated in Figure 2, among the total export value of USD 1.34 billion in 2022, the largest share belonged to HS Code 3307—comprising shaving preparations, deodorants, and similar products—with 24.7%. This was followed by hair preparations (HS Code 3305) with a 22.4% share, and skincare and makeup products (HS Code

3304) with 19.6%. In contrast, other product groups such as essential oils (HS Code 3301) and oral care products (HS Code 3306) had significantly lower shares in total exports.

The fact that high value-added products, particularly skincare and dermocosmetic items, lag behind more traditional categories suggests a limited production capacity based on technology and innovation. The insufficiency of R&D investments and the underdevelopment of branding strategies further reinforce this limitation. Moreover, the concentration of exports in a few key product groups restricts the sector's potential to move upward within the global value chain.

This situation indicates that to enhance its long-term international competitiveness, the Turkish cosmetics industry must prioritize technology-driven transformation, high value-added production, and comprehensive branding initiatives.



Source: TURKSTAT, 2023; Ministry of Trade, 2023.

Figure 2. Distribution of Turkey's Cosmetics Exports by HS Code Categories (2022) Presented as a horizontal bar chart, Figure 2 shows the export shares of cosmetics product groups in 2022. The highest share belonged to shaving preparations and deodorants (HS Code 3307) with 24.7%, followed by hair products (HS Code 3305) with 22.4%, and skincare and makeup products (HS Code 3304) with 19.6%. The chart highlights the sector's focus on traditional and relatively lower value-added product groups, whereas high-technology and innovation-intensive items such as HS Code 3304 remained comparatively limited in share.

3. Geographical Market Dependency and Concentration Risk

Turkey's cosmetics export performance in 2022 reveals a notable degree of geographical dependency in terms of market concentration. Although exports were conducted to a total of 179 countries, the top 10 destinations alone accounted for 55.2% of total exports. This indicates that, despite the appearance of market diversity, the sector is in fact reliant on a limited number of countries. Iraq (8%), the United States (7.2%), and Russia (6.9%) constitute the three largest export markets, followed by countries such as the United Arab Emirates (5.9%), Iran (5.6%), and the Netherlands (5%). European Union member states, including Germany and France, collectively accounted for only around 6.7% of Turkey's total cosmetics exports, suggesting that the sector's orientation toward EU markets remains weaker compared to its focus on the Middle East.

This high degree of market concentration renders Turkey's cosmetics exports vulnerable to global economic fluctuations, political tensions, and bilateral uncertainties. To enhance the sector's sustainable export capacity, the implementation of market diversification policies and the development of target-country-specific strategies have become increasingly essential. Particularly, initiatives aimed at emerging markets such as Africa, Latin America, and Southeast Asia present promising opportunities to mitigate existing concentration risks and reduce dependency on a narrow set of export destinations.

Table 1: Geographical Distribution of Turkey's Cosmetics Sector Exports (2022)

Rank	**Country/ Region**	**Export Share (%)**	**Key Characteristics**
1	Iraq	8.0%	Largest market; high demand for hygiene and personal care products
2	USA	7.2%	Strong demand for soap and perfume products
3	Russia	6.9%	Growing interest in skincare and haircare products
4	United Arab Emirates	5.9%	Strategic re-export hub for regional markets
5	Iran	5.6%	Cross-border trade and regional demand
6	Netherlands	5.0%	Logistics hub for European distribution
7	Germany	3.3%	Limited penetration in dermocosmetics and organic products
8	France	3.4%	Intense competition in the premium segment

Rank	Country/ Region	Export Share (%)	Key Characteristics
Total (Top 10 Countries)	**55.2%**	**Concentration Risk and Need for Diversification**	

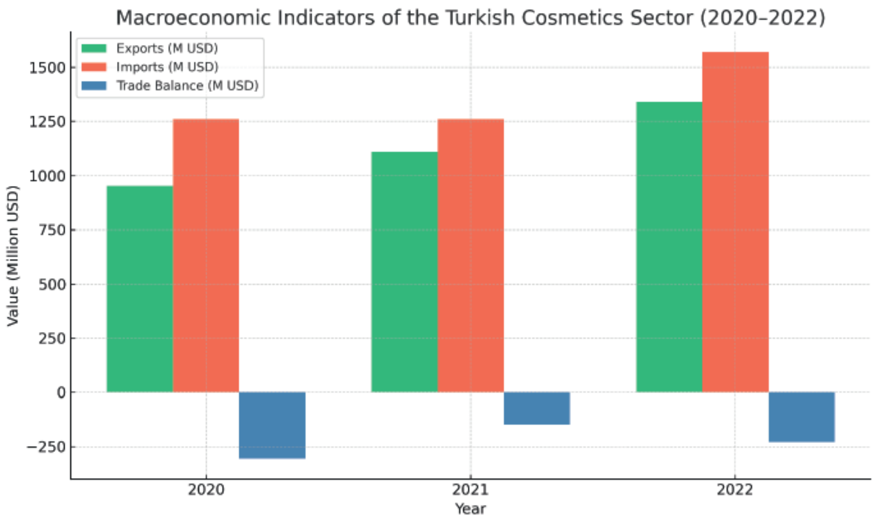
Source: Turkish Statistical Institute (TUIK), 2023; Ministry of Trade of the Republic of Turkey, 2023.

The table highlights Turkey's export concentration in the cosmetics sector, with the top 10 countries accounting for 55.2% of total exports in 2022. Iraq remains the largest market (8.0%), driven by hygiene and personal care products, while the USA (7.2%) and Russia (6.9%) show strong demand for niche categories like perfumes and skincare. The UAE (5.9%) serves as a strategic re-export hub, reflecting Turkey's role in regional supply chains. However, overreliance on key markets (e.g., Iraq, Russia) poses risks, compounded by limited penetration in premium European markets (Germany: 3.3%, France: 3.4%). To ensure sustainable growth, market diversification (e.g., targeting emerging economies) and product innovation (e.g., expanding organic/dermocosmetics lines) are critical strategies. This data underscores the sector's geopolitical dependencies and the need for policies to mitigate concentration risks while leveraging regional trade advantages.

4. Macroeconomic Dynamics and Trade Balance in Turkey's Cosmetics Sector: Analysis of 2022

The 2022 data related to Turkey's cosmetics sector reveal critical findings that underscore the sector's connection to broader macroeconomic dynamics. As reported, in 2021, the Turkish economy experienced an 11% growth in GDP, which was paralleled by a 16.6% increase in cosmetics exports, reaching 1.1 billion USD. In 2022, despite a slowdown in GDP growth to 5.5%, the sector maintained its upward trajectory with a 21% rise in exports, surpassing 1.34 billion USD—demonstrating the sector's economic resilience. This pattern supports the presence of a positive correlation between sectoral expansion and overall economic performance. Specifically, during periods of economic expansion, domestic demand for cosmetics tends to increase, whereas in times of contraction, as seen in the 3.5% GDP decline in 2020 due to the COVID-19 pandemic, the sector's exports fell by 8.3%, dropping to 953 million USD.

Similarly, the 24.5% rise in imports observed in 2022 (reaching 1.57 billion USD) reflects the domestic production capacity's limitations in meeting rising demand and resulted in a net trade deficit of -230 million USD. This trade imbalance can be largely attributed to the sector's reliance on the import of high value-added products, such as perfumes and fragrance compounds. Nevertheless, the recent increase in domestic production of natural and organic products—such as olive oil-based soaps—along with the setting of an ambitious export target of 3.3 billion USD, signals potential for future improvements in trade balance.



Source: Turkish Statistical Institute (TUIK), 2023; Ministry of Trade of the Republic of Turkey, 2023.

Figure 3. Macroeconomic Indicators of Turkey's Cosmetics Sector (2020–2022)

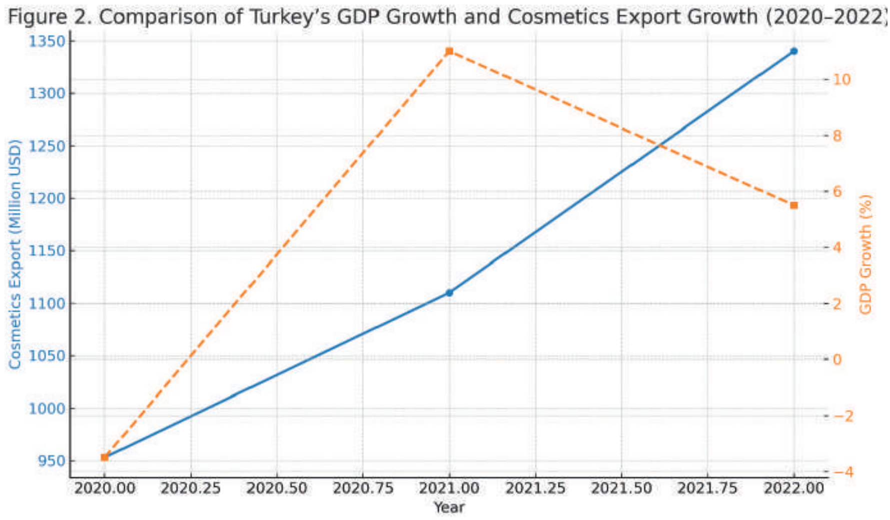
This bar chart presents the export, import, and trade balance figures for Turkey's cosmetics sector between 2020 and 2022. In 2020, under the impact of the COVID-19 pandemic, the sector's export value declined to 953 million USD. In contrast, a steady recovery in the subsequent years led to a significant rise in exports, reaching 1.34 billion USD in 2022, demonstrating the sector's resilience. Meanwhile, imports saw a notable increase of 24.5% in 2022, reaching 1.57 billion USD and pushing the net trade balance to a deficit of -230 million USD. This deficit is primarily driven by the sector's dependence on imported high value-added products, such as perfumes and fragrance compounds. Nonetheless, the growing orientation toward domestic production of natural and organic cosmetics, as well as the sector's new export target of 3.3 billion USD, presents promising opportunities for improving trade balance in the coming years. The chart reflects a positive correlation between general GDP growth and the sector's export performance while also drawing attention to the persistent reliance on imports.

5. Macroeconomic Trends and Export Dynamics in Turkey's Cosmetics Sector (2020–2022)

Correlation analyses between Turkey's GDP growth and the expansion of the cosmetics sector reveal a strong and positive relationship between the country's overall economic performance and sectoral export growth. According to official data published by the Turkish Statistical Institute (TÜİK) and the Ministry of Trade, in 2021, Turkey's GDP expanded by 11%, while cosmetics exports increased by 16.6%, reaching 1.1 billion USD. This parallel upward trend underscores the direct connection between local and international demand for cosmetics products and periods of economic expansion.

Conversely, the economic contraction caused by the COVID-19 pandemic in 2020 led to a 3.5% decline in GDP and an 8.3% drop in cosmetics exports, which fell to 953 million USD. In 2022, despite a slowdown in GDP growth to 5.5%, cosmetics exports continued to rise, increasing by 21% and reaching 1.34 billion USD. This divergence points to a growing global interest in Turkish cosmetics and indicates the sector's increasing resilience and adaptive capacity.

These findings reinforce the strategic significance of the cosmetics industry as both a reflection of broader macroeconomic trends and a driver of those trends. Furthermore, considering the volatility of domestic economic indicators, the sustainability of this export performance highlights the need for structural reforms to ensure long-term sectoral stability and competitiveness.



Source: Turkish Statistical Institute (TÜİK), 2023; Republic of Turkey Ministry of Trade, 2023.

Figure 4. Correlation Between GDP Growth and Cosmetics Export Growth in Turkey (2020–2022)

This dual-axis line graph illustrates the relationship between Turkey's annual GDP growth rate and the export growth rate of the cosmetics sector between 2020 and 2022. In 2021, both indicators experienced a sharp rise, with GDP increasing by 11% and cosmetics exports growing by 16.6%. In contrast, during the pandemic-driven contraction of 2020, GDP shrank by 3.5% and cosmetics exports declined by 8.3%. In 2022, despite a deceleration in GDP growth to 5.5%, cosmetics export growth accelerated to 21%. This trend suggests a rising global demand for Turkish cosmetics and indicates that the sector may be gradually decoupling from domestic economic cycles.

6. Structural Position of Turkey's Cosmetics Sector: SWOT-Based Findings

Based on 2023 data, a structural analysis of Turkey's cosmetics sector reveals a competitive yet fragile framework shaped by the interplay between internal capacity and external market conditions. According to the 2023 Cosmetics Sector Report published by the Ministry of Trade, the sector's strengths include Turkey's strategic geographical position close to both Europe and the Middle East, its relatively low production costs, and a young, dynamic domestic market.

However, several fundamental structural challenges persist. These include insufficient investment in R&D, limited international brand recognition, and a significant dependence on imports—particularly in high value-added product categories such as perfumes and dermocosmetics. These weaknesses are among the main factors preventing the sector from moving higher within the global value chain.

From an opportunity perspective, the growing global demand for natural, organic, and vegan cosmetics presents important growth potential for the sector. The rapid expansion of e-commerce and direct-to-consumer (D2C) sales models, especially in the post-pandemic period, has also enhanced market access and competitiveness for small and medium-sized enterprises (SMEs).

Nonetheless, the sector faces several critical threats. Exchange rate volatility, dependency on imported raw materials, and difficulties in complying with international certification standards (e.g., ISO 22716, GMP) constitute significant barriers for exporting firms. Additionally, frequent changes in national-level regulations and uncertainties in implementation hinder the integration of domestic producers into global supply chains.

Table 1: SWOT Analysis of Turkey's Cosmetics Sector (Updated)

Strengths	**Weaknesses**	**Opportunities**	**Threats**
Strategic geographic location (proximity to Europe and the Middle East)	Insufficient R&D investments	Rising global demand for natural, organic, and vegan products	Challenges in international certifications (ISO, GMP, etc.)
Competitive production costs	Low international brand recognition	Expansion of e-commerce and D2C (direct-to-consumer) sales channels	Exchange rate volatility
Diverse product portfolio and production capacity	Dependency on imports (especially high value-added products)	Growing interest in eco-friendly products among younger demographics	Global economic uncertainties and inflationary pressures
Young and dynamic domestic market	Lack of sectoral quality standardization	Expansion opportunities in emerging markets (Africa, Asia)	Rising raw material and logistics costs
Supportive impact of cosmetic tourism and aesthetic healthcare services	Inadequate digital brand management	Sustainability funds and incentives under the Green Deal compliance process	Frequent changes in legal regulations and implementation uncertainties

Sources: Turkish Statistical Institute (TUIK), 2023; Ministry of Trade of Turkey, 2023.

Turkey's cosmetics sector stands out in the global marketplace with its strategic location, competitive cost structure, and wide product portfolio. However, weaknesses such as insufficient R&D, import dependency, and low international brand recognition continue to limit the sector's potential.

Rising demand for natural and vegan products, the growth of e-commerce, and access to emerging markets in Africa and Asia offer promising opportunities for growth. Yet, threats such as exchange rate volatility, certification challenges, and global economic uncertainty pose obstacles to sustainable development.

Content analyses supported by qualitative data reveal that sectoral policy documents frequently emphasize themes such as "export growth," "value-added production," "branding," and "internationalization." According to the Ministry of Trade's 2023 Cosmetics Sector Report, key strategic goals include increasing the sector's exports to 2 billion USD by 2025 and achieving global brand recognition for at least three domestic companies. These goals reflect the sector's long-term strategic vision and serve as key reference points for public policy formulation.

In conclusion, the 2020–2022 period can be viewed as a phase of both growth and structural transformation for the Turkish cosmetics sector. The data suggest that while the sector has increased its competitiveness in global markets, structural reforms are necessary to ensure the sustainability of this progress.

DISCUSSION AND CONCLUSION

The findings obtained in this study reveal that Turkey's cosmetics sector demonstrated a remarkable growth performance between 2020 and 2022. The increase in export values from 953 million USD to 1.34 billion USD within three years and the achievement of a compound annual growth rate (CAGR) of 18.7% indicate that the sector remained resilient despite pandemic conditions and retained its competitiveness in the global market. The steady upward trend in foreign trade figures, in particular, highlights both the sector's growing influence in regional markets and its expanding production capabilities.

The results also show that Turkey's cosmetics exports are concentrated in a limited number of countries. While exports to markets such as Iraq, Germany, the United Arab Emirates, and Saudi Arabia account for over 50% of total exports—demonstrating Turkey's effectiveness in both emerging and developed markets—this also indicates the need for market diversification. These findings align with the Ministry of Trade's (2023) evaluation of the sector's "Middle East and Europe-oriented structure." Moreover, calculations of export concentration indices reinforce the conclusion that the current structure is unsustainable and underline the urgency of identifying alternative markets.

In terms of product-based evaluation, it was found that shaving and deodorant products accounted for 24.7% of total exports, whereas dependence on imports for high value-added products such as skincare and makeup reached up to 44%. This demonstrates that while Turkey has a strong production infrastructure for lower value-added products, it faces deficiencies in the production and branding of high value-added goods, particularly in the dermocosmetic segment. These findings are consistent with the observations of Erkan and Sarıoğlu (2022), who emphasized the lack of branding and R&D in developing economies. The fact that R&D expenditures in Turkey amount to only 1.8% of company revenues is considered one of the main limitations to the sector's innovation capacity.

According to the SWOT analysis included in the study, the sector's key strengths include a young and dynamic population, strategic geographic location, and diverse product range. On the other hand, major weaknesses include low brand recognition, dependency on imports, and lack of standardization within the sector. Meanwhile, rising global demand for natural and organic products, the spread of e-commerce, and entry into new markets offer important opportunities. However, exchange rate volatility, certification requirements, and global competition continue to pose serious threats to the sustainability of the sector.

In light of these findings, a number of strategic recommendations have been developed to ensure the sustainable growth of the sector and to enhance its global competitiveness. First, a "Turkish Cosmetics Brands" program should be launched to improve the visibility of domestic brands in international markets. R&D incentives should be increased to promote the development of high value-added product lines, especially in the areas of organic, dermatological, and vegan cosmetics. Targeted export strategies should be formulated for underutilized markets such as the United States and the Far East, and e-export channels should be expanded to support direct-to-consumer (D2C) models. Furthermore, adapting production processes to sustainability criteria, such as biodegradable packaging and ISO 22716 standards in alignment with the European Green Deal, may offer significant competitive advantages.

Naturally, this study has certain limitations. Most notably, the lack of officially published data for 2023 and beyond restricts the ability to analyze more recent trends. Additionally, the absence of firm-level profitability data prevents a more detailed assessment of cost-efficiency and productivity. To address these limitations, future studies should incorporate firm-based qualitative research and examine the impact of technologies such as digital transformation, artificial intelligence, and the Internet of Things (IoT) on production processes in greater depth.

In conclusion, as of the 2020–2022 period, Turkey's cosmetics sector stands out as a rapidly growing industry with increasing effectiveness in international markets. However, ensuring the sustainability of this growth will require strategic investments in branding, R&D, sustainability, and market diversification. This study not only analyzes the current state of the sector but also aims to offer a roadmap for policymakers and industry stakeholders.

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