Chapter 6

Multivariate Anomaly Mapping in Video Games: A Mahalanobis Distance Approach

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Abstract

This paper addresses the detection of anomalous video games by analyzing disparities between key variables, including the game's price, the number of recommendations, the percentage of positive and negative reviews, and the estimated owners. The research aims to develop a sophisticated model that identifies games significantly deviating from expected patterns based on multidimensional relationships between these variables. Advanced statistical techniques were employed for outlier detection, including *z*-scores, which quantify deviations from the mean, interquartile ranges (IQR), which pinpoint extreme values within the data distribution, and Mahalanobis distance, which allows for anomaly detection by incorporating correlations between variables and multidimensional differences. Utilizing a covariance matrix, this method ensures precise identification of outliers, even in complex datasets with multiple correlated variables.

The detected anomalies were confirmed through appropriate visualization techniques, which enable the clear identification of exceptional cases and patterns that deviate from the expected distributions. These visualizations deepen the understanding of the anomalies, allowing for the formulation of critical questions regarding the credibility and balance of attributes such as price, ratings, recommendations, and ownership, as well as potential latent market anomalies.

Preliminary results suggest the existence of several notable outliers, including games with exceptionally high prices coupled with poor ratings, and games that have an extraordinarily high number of recommendations despite low average playtime. These findings point to potentially disruptive patterns in market dynamics, potentially stemming from marketing manipulation or inadequate recommendation systems that fail to reflect the true quality of the

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game. Identifying these anomalies lays the groundwork for further research aimed at reducing recommender bias and optimizing product rating systems, focusing on objective rather than subjective product characteristics.

INTRODUCTION

Analyzing and identifying anomalies in complex datasets is essential for uncovering structural deviations within digital markets. The digital distribution infrastructure and recommendation pipelines reflect deeper networked mechanisms that influence visibility and consumption (Kurose and Ross, 2016). In the context of the video game industry, a rapidly expanding segment of the software domain, the abundance of publicly available data facilitates rigorous quantitative analysis of user behavior and market dynamics. Conventional models for classifying games typically rely on categorical attributes such as genre, platform or publisher, while evaluative scores are frequently based on singular indicators, including average ratings or total download counts. These univariate approaches fail to capture the multidimensional interdependencies that collectively define a product's market positioning.

This study aims to detect statistically atypical video games through a multivariate framework that incorporates several interrelated variables: price, number of recommendations, proportion of positive and negative reviews, and the number of users who have reported owning the game. While standard outlier detection techniques such as z-scores and interquartile ranges were employed for reference, the primary analytical emphasis was placed on Mahalanobis distance. This method enables the identification of multidimensional anomalies by incorporating the covariance structure of the data, thereby allowing for the detection of unusual variable constellations that remain obscured in univariate analyses.

The Mahalanobis distance enables quantification of each observation's deviation from the multivariate centroid, thereby facilitating the robust detection of structurally deviant instances. The identified anomalies were examined in relation to market dynamics, recommendation frequency and user interaction, with visualizations employed to verify and interpret the findings. Particular emphasis was placed on games that commanded unusually high prices despite negative user ratings, as well as those with disproportionately high recommendation counts coupled with low engagement metrics, patterns that may suggest distortions in rating systems or irregularities in algorithmically generated recommendations. This paper aims to construct and validate an analytical framework for detecting multidimensional anomalies in video game data, thereby identifying products that substantially diverge from expected behavioral and evaluative norms. The proposed approach supports the objectification of market value estimations, mitigates algorithmic bias in recommendation systems and promotes the development of more transparent models for the valuation of digital products. The findings are transferable to broader applications involving consumer behavior analysis, market strategy optimization and the quantitative assessment of commercial performance.

The starting point for this approach is recent research examining the theoretical foundations and practical applications of Mahalanobis distance and related methods for anomaly detection across various domains. The effectiveness of Mahalanobis distance in identifying anomalies has been evaluated with particular emphasis on out-of-distribution examples and adversarial inputs (Kamoi and Kobayashi, 2020). A variant of the method based on the marginal distributions of features without requiring class labels was introduced, demonstrating competitive performance relative to more complex alternatives. The analysis confirmed that multidimensional features, which are frequently overlooked in traditional classification models, can play a critical role in the reliable detection of data irregularities.

Building on a similar principle, the MDX framework was developed for anomaly detection in deep reinforcement learning environments (Zhang et al., 2024). State distributions were modeled using class-conditional Gaussian functions, and deviations were assessed through multivariate statistical tests. The method was enhanced with robust extensions and compliant algorithms, and validated on examples from video games, simulation environments and autonomous driving systems, underscoring its practical utility in complex interactive settings.

Further advancement in deep learning models incorporating the Mahalanobis component was contributed by Pinto et al. (2021), who proposed a hybrid anomaly detection approach combining Mahalanobis distance in the latent space of autoencoders with generative adversarial networks. Rather than relying on direct reconstruction errors, the method estimated deviations based on the distance of latent vectors from a multivariate Gaussian kernel, thereby reducing dependence on model nonlinearities and improving detection accuracy. The approach demonstrated consistent performance across temporal and sensor datasets, with the Mahalanobis component enhancing the robustness of multidimensional anomaly detection.

A method for video anomaly detection based on the selection of representative normal samples facilitated more stable modeling of the reference distribution and reduced false-positive detections (Wu et al., 2021). This approach relies on the dynamic adjustment of a set of normal samples, which improved outlier detection quality in visually evolving sequences. An approach for online detection and localization of multivariate anomalies in high-dimensional data employs an unsupervised k-NN technique for sequential outlier identification (Mozaffari et al., 2022). The method demonstrated asymptotic optimality and scalability across various domains such as IoT and video surveillance and allows for the isolation of specific dimensions responsible for anomaly detection. In a related comparative study, Dobos et al. (2023) conducted a systematic comparison between traditional statistical tests and modern machine learning-based anomaly detection methods in the context of gross error detection. Their findings confirmed that multivariate methods, such as One-Class SVM and IQR, can effectively identify anomalous patterns even in synthetic datasets with engineered noise and bias. These results support the applicability of unsupervised detection techniques in other high-dimensional domains, including user and product data analytics. Complementary evidence was provided by Lin and Li (2024), who investigated the influence of distance metrics in segmentation and detection tasks. They demonstrated that the choice of k-value in k-NN significantly affected performance when applied to industrial visual data. Their results emphasized the importance of metric selection and reinforced the rationale for employing Mahalanobis distance in the multivariate analysis of game-related data.

Azizi and Zaman (2023) introduced a method for automatic bug detection in video games using LSTM networks. By analyzing gameplay logs and temporal patterns, their model identified irregular sequences associated with non-player character behavior and object interactions. This deep learning-based approach underscored the broader relevance of anomaly detection in the context of game testing and complemented conventional statistical techniques. Zhang et al. (2021) developed a lightweight framework for anomaly detection in deep reinforcement learning (DRL). Their method relied on auxiliary prediction tasks to capture discrepancies between expected and observed agent behavior during training. It was evaluated across standard DRL environments and demonstrated the capability to detect anomalous states without requiring labeled anomaly data.

The use of the model-agnostic meta-learning (MAML) framework combined with the Swin transformer for spatial feature extraction enables rapid adaptation to novel scenarios and high efficiency in distinguishing anomalies within video sequences (Singh et al., 2025). The approach achieved an AUC of 0.91 on the MSAD dataset, illustrating how multisituational anomaly detection strategies can be adapted to complex and variable environments. Wang et al. (2025) analyzed deep anomaly detection methods in multidimensional time series, highlighting the strengths and limitations of various architectures, including autoencoders, generative adversarial networks, and transformers. Particular emphasis was placed on challenges such as distributional variability, limited interpretability, and the necessity of integrating statistical measures such as Mahalanobis distance to enhance detection accuracy and robustness. An empirical approach was used to analyze a series of two online titles to identify anomalies in player behavior. Four unsupervised methods (LOF, KDE, K-means, GMM) were used to identify deviant users (Dinh et al., 2016). The evaluation was conducted using artificial and real data from the JX2 and Chan games. The performance analysis was based on detection accuracy, number of anomalies identified, and execution time, and showed the advantage of parametric models in terms of speed and comparable accuracy compared to non-parametric methods.

Irvan et al. (2024) applied LSTM networks to anomaly detection in the context of the e-sports game CS:GO, focusing on player movement patterns. The model was trained in normal sequences, adding synthetic anomalies such as teleportation and bot behavior. Experiments have shown that LSTM reliably detects deviations in real time, with an accuracy of over 90 percent under standard conditions, but with a drop in performance with a high proportion of anomalous patterns.

While prior research confirms the versatility of Mahalanobis distance in anomaly detection across a range of domains, its application to market-facing behavioral data remains underexplored. The present study addresses this gap by introducing a multivariate framework for anomaly identification in video games, explicitly designed to balance statistical precision with interpretability and market relevance.

METHODOLOGY

The analytical framework of this research is based on multidimensional anomaly detection using Mahalanobis distance, with the integration of auxiliary statistical mechanisms such as z-score and interquartile range (IQR) to increase robustness in identifying deviant instances in multidimensional user space (Hair et al., 2019). The entire approach was conducted in a quantitative regime, applying linear algebraic methods and empirical covariance matrices.

Variables and Standardization

To analyze latent patterns of similarity between video games, six multidimensional variables were selected to capture essential aspects of pricing, user engagement, and rating distribution. The selection criteria were based on the potential of each variable to reflect user behavior and market positioning. All data was obtained from the publicly accessible Steam Games Dataset available on the Kaggle platform.

The data sample comprises $n = 1000$ unique video games, each described by six variables as follows:

- price (x_1): the official retail price of the game in euros, based on the most recent listed value in the store database,
- number of recommendations (x_2): the total number of user-submitted positive endorsements recorded on the platform,
- percentage of positive reviews (x_3): the ratio of positive user evaluations to the total number of reviews, expressed as a percentage,
- percentage of negative reviews (x_4): the corresponding share of negative user evaluations, also expressed as a percentage,
- estimated owners (x_5): the approximated number of individual users who own the game, based on publicly available player count ranges and platform-specific estimation methods.

Each instance $x_i \in R^5$ represents a row of the data matrix $\boldsymbol{X} \in R^{nx5}$,

To ensure metric homogeneity and eliminate the effects of differing variable scales, z-score standardization was applied to each variable:

$$z_{ik} = \frac{x_{ik} - \mu_k}{\sigma_k}$$

for $i = 1,\ldots,$n and $k = 1,\ldots,5$, where:

$$\mu_k = \frac{1}{n}\sum\nolimits_{i=1}^{n} x_{ik}$$

enotes the empirical mean of variable x_k, and the empirical standard deviation is defined as:

$$\sigma_k = \sqrt{\frac{1}{n-1}\sum\nolimits_{i=1}^{n}\left(x_{ik} - \mu_k\right)^2}$$

These variables were chosen based on their ability to reflect user valuation, engagement intensity, and perceived quality, offering a multidimensional perspective of game performance. The resulting matrix of standardized values is denoted by $\boldsymbol{Z} \in R^{nx5}$, where $\boldsymbol{z}_i$ is the i-th row vector.

Empirical covariance matrix

The covariance matrix captures the linear interdependencies among standardized variables, serving both as the mathematical foundation for Mahalanobis distance computation and as a central construct in multivariate statistical analysis (Hair et al., 2019).

Based on the standardized matrix $\boldsymbol{Z}$, the empirical covariance matrix $\Sigma \in R^{5x5}$ was computed as:

$$\boldsymbol{\Sigma} = \frac{1}{n-1} \boldsymbol{Z}^{\mathrm{T}} \boldsymbol{Z}$$

If the matrix Σ exhibits numerical instability or is close to singular, regularization is applied following the Tikhonov principle:

$$\boldsymbol{\Sigma}_{\lambda} = \boldsymbol{\Sigma} + \lambda \boldsymbol{I}_5$$

where $\lambda \in R^{+}$ denotes a small regularization parameter (tipically $\lambda = 10^{-2}$), and $\boldsymbol{I}_5$ is the 5×5 identity matrix.

Mahalanobis Distance

The Mahalanobis distance for the i-th instance is defined in quadratic form as:

$$D_i^2 = \left(\boldsymbol{z}_i - \boldsymbol{\mu}\right)^T \boldsymbol{\Sigma}^{-1} \left(\boldsymbol{z}_i - \boldsymbol{\mu}\right)$$

Given that all vectors are standardized $(\boldsymbol{\mu} = \mathbf{0}_5)$, the expression simplifies to:

$$D_i^2 = \boldsymbol{z}_i^T \boldsymbol{\Sigma}^{-1} \boldsymbol{z}_i$$

For the entire set of n instances, the vector of Mahalanobis distances is obtained as the diagonal of the matrix product:

$$D^2 = \operatorname{diag}\left(\boldsymbol{Z}\boldsymbol{\Sigma}^{-1}\boldsymbol{Z}^T\right)$$

This vector of squared distances serves as the criterion for deviation detection. In this context, the formulation enables robust anomaly identification by highlighting instances that are structurally distant from the global feature centroid, while accounting for both scale and correlation effects.

By incorporating the inverse covariance matrix as a scaling transformation, Mahalanobis distance generalizes the Euclidean metric to ellipsoidal contours aligned with the data's variance structure, enabling anisotropic sensitivity to deviations along correlated dimensions.

Statistical Threshold and Deviation Classification

Outlier instances are identified based on the squared Mahalanobis distance compared to the threshold derived from the chi-squared distribution with six degrees of freedom. An instance *i* is considered an outlier if $D_i^2 > x_{1-\alpha}^2(5)$. For the significance level α=0.01, the threshold value is $\chi_{0.99}^2(5) \approx 15.086$, and for α = 0.05 it is $\chi_{0.95}^2(5) \approx 11.070$.

This statistical decision rule leverages the asymptotic properties of the Mahalanobis distance under multivariate normality, ensuring that the rejection region corresponds to a predefined probability mass in the tail of the distribution. By anchoring deviation detection to a chi-squared quantile function, the methodology ensures a theoretically justified and probabilistically interpretable classification boundary, independent of specific distributional idiosyncrasies in the data.

Additional Robustness Criteria

To enhance the reliability of anomaly detection and identify borderline cases, supplementary statistical criteria have been applied:

- *Z-score criterion*: an instance is marked as a potential outlier if ∃ k ∈ {1,…,5} such that $|z_{ik}| > 3$.
- *IQR criterion*: for each variable x_k threshold values are defined as follows:

$$\text{lower treshold}_k = Q_1^{(k)} - 1.5 \cdot IQR_k$$
$$\text{upper treshold}_k = Q_3^{(k)} - 1.5 \cdot IQR_k$$

where $IQR_k = Q_3^{(k)} - Q_1^{(k)}$, and $Q_1^{(k)}\, i\, Q_3^{(k)}$ denote the first and third quartiles of the variable *xk*. Each instance whose attribute lies outside these bounds is additionally labeled as a potential outlier.

These supplementary criteria ensure that anomalies with marginal Mahalanobis distances, but extreme univariate characteristics are not overlooked. Moreover, by incorporating both standardized deviation (*z*-score) and distributional spread (IQR), the model achieves a dual-layered safeguard against the misclassification of edge cases that may evade detection under multivariate-only metrics.

Implementation in Python

All data processing phases were carried out using the Python programming language (version 3.11), along with the following libraries:

- pandas: for structured data import, filtering, user-based grouping, and review aggregation (*pandas.read_csv*, *DataFrame.loc*, *groupby*, *dropna*),
- numpy: for numerical processing of vectors and matrices, including mean calculation, covariance estimation, and linear algebra operations (*numpy.array*, *numpy.mean*, *numpy.std*, *numpy.dot*),
- scipy.stats: for statistical computations such as z-scores, interquartile ranges, and chi-squared quantiles (*zscore*, *iqr*, *chi2.ppf*),
- scipy.spatial.distance: for computing Mahalanobis distances in multidimensional space (*mahalanobis*),
- matplotlib.pyplot: for creating visual representations of distributions, clusters, and deviations (*scatter*, *hist*, *subplots*, *tight_layout*),
- seaborn: for enhanced statistical visualization, including heatmaps, boxplots, and cluster maps (*heatmap*, *boxplot*, *clustermap*),
- scikit-learn: for feature standardization (*StandardScaler*), robust scaling (*RobustScaler*), and outlier evaluation (*StandardScaler*, *RobustScaler*).

The implementation strategy draws on practical principles outlined in recent literature on applied deep learning (Howard and Gugger, 2020; Elgendy, 2022).

All variables were standardized using the z-score method to ensure metric comparability during distance calculations. The Mahalanobis values per instance are stored in the variable *md_scores*, while outlier thresholds derived from the empirical χ^2 distribution are stored in *md_thresholds*. The classification of instances into regular and anomalous is saved in *md_outliers*. The full analysis was conducted in the script *mahalanobis_detection.py* and additionally documented in the Jupyter notebook *outlier_mapping.ipynb*, which includes visualizations of distributions, heatmaps, and distance-based outlier mappings.

This methodological framework establishes a rigorous statistical basis for identifying structurally atypical instances, whose empirical characteristics and distributional deviations are further explored in the following section.

RESULTS AND DISCUSSION

This chapter presents the outcomes of the multivariate deviation analysis applied to the video game dataset. The primary analytical framework is based on the Mahalanobis distance, supplemented by additional robustness metrics. The results are structured according to the behavioral variables included in the model, with separate sections detailing the inter-variable relationships, empirical distributions, and classification of instances based on deviation thresholds. Visual representations are provided to complement the numerical findings and to enhance the interpretability of detected anomalies.

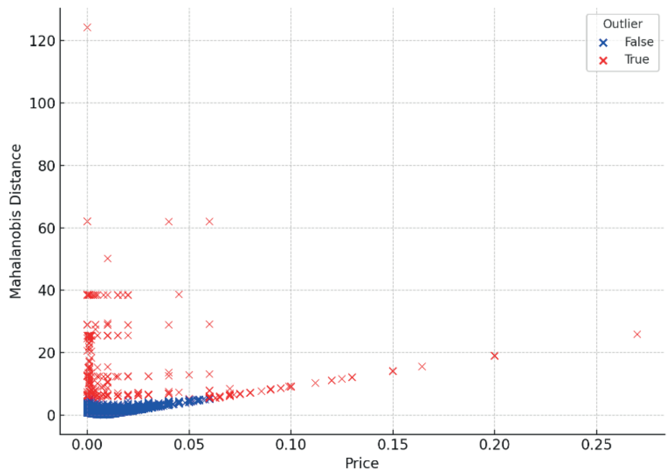

Figure 1. Multivariate Outlier Detection by Mahalanobis Distance in Relation to Game Price

The scatter plot in Figure 1 displays the relationship between video game price (x-axis) and Mahalanobis distance (y-axis), with each point representing one game. Instances marked in red correspond to statistical outliers in the multivariate feature space. The majority of games are concentrated in the lower left region of the plot, indicating lower prices and proximity to the multivariate centroid. A smaller subset of games exhibits high Mahalanobis distances, with some also characterized by exceptionally high prices. These instances suggest deviations from the general pattern, potentially reflecting atypical combinations of attributes, such as elevated cost coupled with uncommon features or low user evaluations. The observed separation indicates that these games differ not only in price, but also across multiple behavioral dimensions, thereby qualifying as potential anomalies requiring further qualitative analysis.

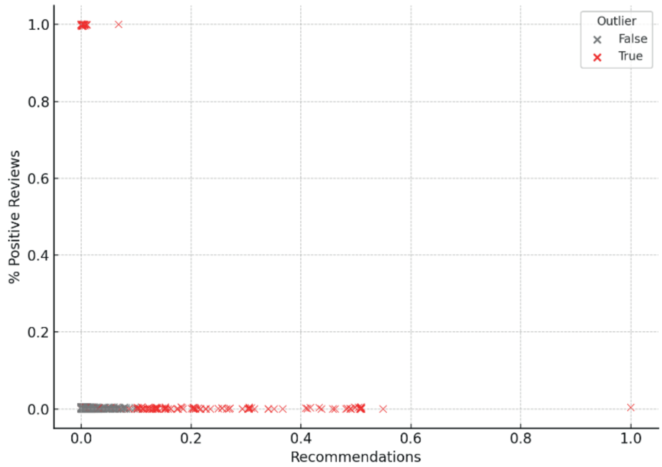

Figure 2. Outlier Distribution by Recommendation Frequency and Positive Review Ratio

Figure 2 shows the relationship between the number of recommendations (x-axis) and the proportion of positive ratings (y-axis) for each game, with outliers marked based on Mahalanobis distance (red markers). The majority of games are clustered in the lower left quadrant, indicating low values for both metrics. However, several anomalies deviate from this trend. Certain titles display a high number of recommendations despite a low proportion of positive ratings, which may suggest atypical marketing practices or manipulation of recommendation algorithms. Conversely, some games exhibit high approval rates but a minimal number of recommendations, potentially reflecting niche targeting or a narrowly defined genre. The application of Mahalanobis distance allowed for the detection of such inconsistencies, which would remain obscured using univariate methods. These findings highlight the need for further contextual investigation to explore underlying marketing, sales, or content-related factors.

Figure 3. Correlation Matrix of All Five Standardized Input Variables

The heatmap in Figure 3 visualizes the correlation matrix of all five standardized input variables. The only strong and statistically expected correlation is a perfect negative relationship between the percentage of positive and negative ratings (-1.00), as these two measures are mathematically complementary. All other variables, including price, number of recommendations, and estimated number of owners, show very low or negligible correlations with the remaining variables, suggesting an absence of linear dependencies within the dataset. This structure supports the application of a multivariate method such as Mahalanobis distance, since univariate metrics are inadequate for capturing complex deviations. Each variable contributes unique informational value to the model without introducing substantial multicollinearity.

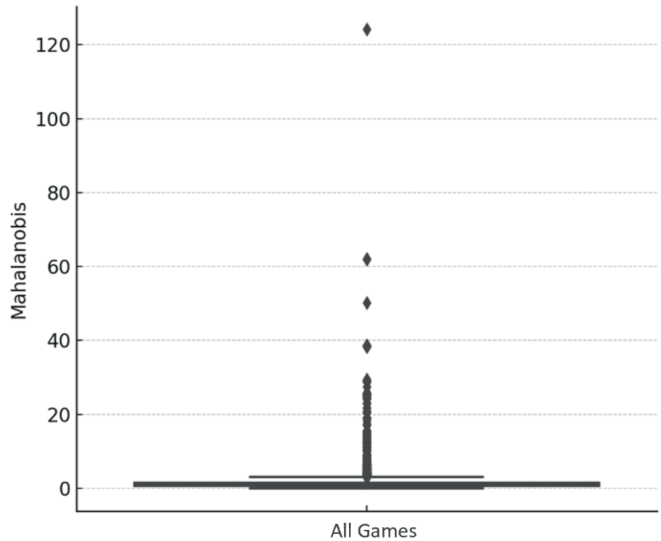

Figure 4. Distribution of Mahalanobis Distances

Figure 4 shows a boxplot of Mahalanobis distances, revealing an asymmetric distribution with a pronounced right tail. Most instances are clustered near the lower quartile, indicating low distance values and alignment with the average multivariate patterns. However, several extreme values clearly deviate from the rest, with the maximum exceeding 120. These cases represent statistically significant anomalies when all variables are considered jointly. The shape of the distribution confirms the effectiveness of the Mahalanobis metric in isolating atypical entities within a multidimensional space.

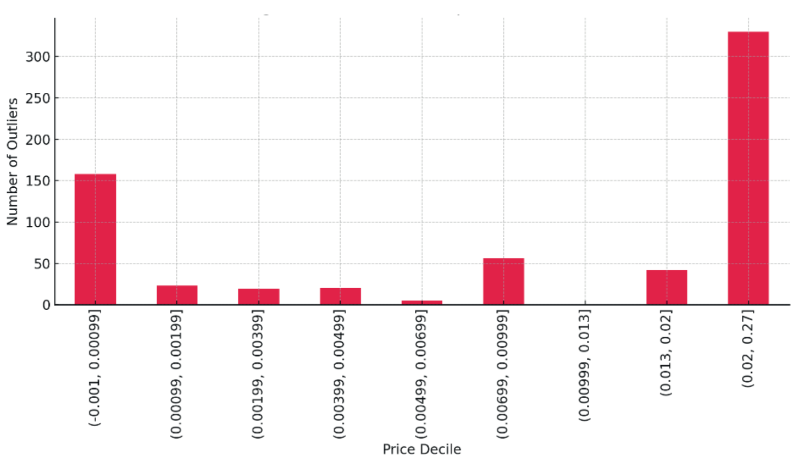

Figure 5. Distribution of Outliers Across Price Deciles

The histogram by price decile in Figure 5 indicates that extreme price values are most frequently associated with statistical anomalies. The highest number of outliers occurs in the top decile (0.20–0.27), representing the most expensive games, and in the bottom decile (0.001–0.00099), corresponding to the cheapest titles. In contrast, middle price ranges show a markedly lower incidence of deviations, suggesting greater consistency in market positioning and user feedback. This distribution supports the hypothesis that games with extreme prices are more statistically unstable within the multidimensional feature space.

Overall, the analysis confirms that multidimensional methods such as Mahalanobis distance are effective in detecting statistically atypical patterns, particularly for games that diverge in terms of pricing or user perception. These findings offer a robust foundation for further investigation of market anomalies and potential mechanisms of visibility or reputation manipulation.

CONCLUSION

This study demonstrates that multiple anomaly detection is an effective approach for identifying video games that deviate from expected behavioral patterns and typical attribute values. By applying Mahalanobis distance within a standardized feature space, the analysis detects complex deviations that would not be apparent using simpler metrics. The identified anomalies include games with disproportionately high prices relative to low user ratings and titles with an unusually high number of recommendations despite overall poor scores. Such inconsistencies suggest potential distortions in market visibility or recommendation mechanisms.

The findings underscore the limitations of relying solely on conventional popularity and rating metrics and confirm the added value of objective, data-driven evaluation models. Visualizations further support the interpretation of these deviations and reinforce the robustness of the analytical procedure. Additionally, the results indicate that anomaly detection methods can enhance transparency in digital markets and recommendation systems by revealing items that diverge from normative patterns.

Future extensions of this framework could incorporate temporal usage patterns, session-level behavior, or genre-based features. Transitioning from unsupervised to supervised models may also improve classification accuracy and result interpretability. The methodology presented here thus provides a repeatable and adaptable basis for the systematic detection of anomalies in interactive digital content.

The Python script used for the analysis is available on request.

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Conflict of Interest

The author has declared that there is no conflict of interest".

Author Contributions

All aspects of the study, including conceptualization, methodology, analysis, and writing, were carried out by the sole author.